

Elements of Statistical SPICE Models

N. Lu, J. Watts, S. Springer

IBM

*Semiconductor Research & Dev. Center
Burlington, Vermont, USA*

Outline

- **Requirements for Statistical SPICE Models**
- **Elements in Statistical SPICE Models**
 - 1. Distinguishing Monte Carlo vs. corner simulations
 - 2. Modeling of asymmetric statistical distributions
 - 3. Treatment of many process statistical distributions in skewed/corner simulations
 - 4. Modeling the combined effects of chip-mean (systematic) and across-chip (random) variations
- **Summary**

Requirements for Statistical SPICE Models

- **Support both Monte Carlo (MC) and skewed/corner simulations**
 - Best also support Across-Chip Variation (ACV) MC simulations at a chip-mean corner
- **Able to handle severe asymmetric distributions**
 - Best also to handle an arbitrary distribution which can be symmetric, moderately asymmetric, or severely asymmetric
- **Provide both easy skewing (few skewing parameters) and full skewing (many skewing parameters; for optimal corner)**
- **Correctly combine global and local variations**

1. Distinguish MC vs. Corner Simulations

- **A typical method:**

```
.param mc_sw = 0      $ 0 or 1  
+ rs_f = rs_n + rs_3sig * (mc_sw * agauss(0,1,3)  
                          + (1 - mc_sw) * cor_res/3)
```

- **Problem: Switch setting and actual netlist's MC or corner simulations are two different things. No guarantee that the two will be the same**

- **Another typical method:**

```
+ .param cor_res = 0.0  
+ rs_f = rs_n + rs_3sig * agauss(cor_res/3, 1, 3)  
and ask users not to run MC with cor_res  $\neq$  0
```

- **Problem: Does not prevent model's misuse**

Element 1. Our Solution

- **Automatic detection of MC vs. skewed/corner simulations**

```
.param mc1 = agauss(0, 1, 3)
+         mc2 = agauss(0, 1, 3)
+         mc3 = agauss(0, 1, 3)
+ is_mc = 'mc1 != 0 || mc2 !=0 || mc3 !=0'
...
+ p1_f = 'is_mc ? p1_mc : p1_skewing'
+ p2_f = 'is_mc ? p2_mc : p2_skewing'
```

- **There are other automatic detection methods. For example:**

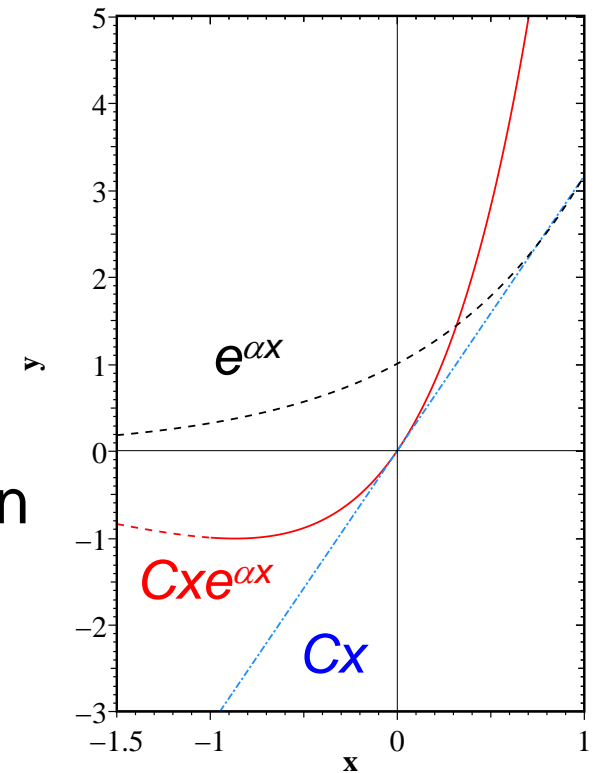
```
+ is_mc = 'mc1 != mc2 || mc2 != mc3'
```

2. Modeling asymmetric distributions

- **Solution method: Use a mapping relation to transfer a symmetric distribution to an asymmetric one**
- **Requirements:**
 - Mapping relation be monotonic between 3σ lower and 3σ upper bounds
 - Be able to model a severe asymmetric distribution
- **Best solution:**
 - Can handle an arbitrary distribution (symmetric, moderately asymmetric, or severely asymmetric) at run time
 - Usage: Statistical SPICE models for supporting parasitic R and C extraction tools (PEX tools)
 - Distributions of parasitic R and C are specified in a SPICE netlist by a PEX tool after the models are built

Element 2. Existing solutions

- $y = x^2$, $y = x^3$, $y = e^x$, etc.:
 - There is an upper limit on $(\max - \text{nom})/(\text{nom} - \min)$
 - Can not handle both symmetric and asymmetric distribution at run time
- $y = Cxe^{ax}$, etc.:
 - Can handle both symmetric and asymmetric distributions
 - But is not always monotonic between min and max of a distribution

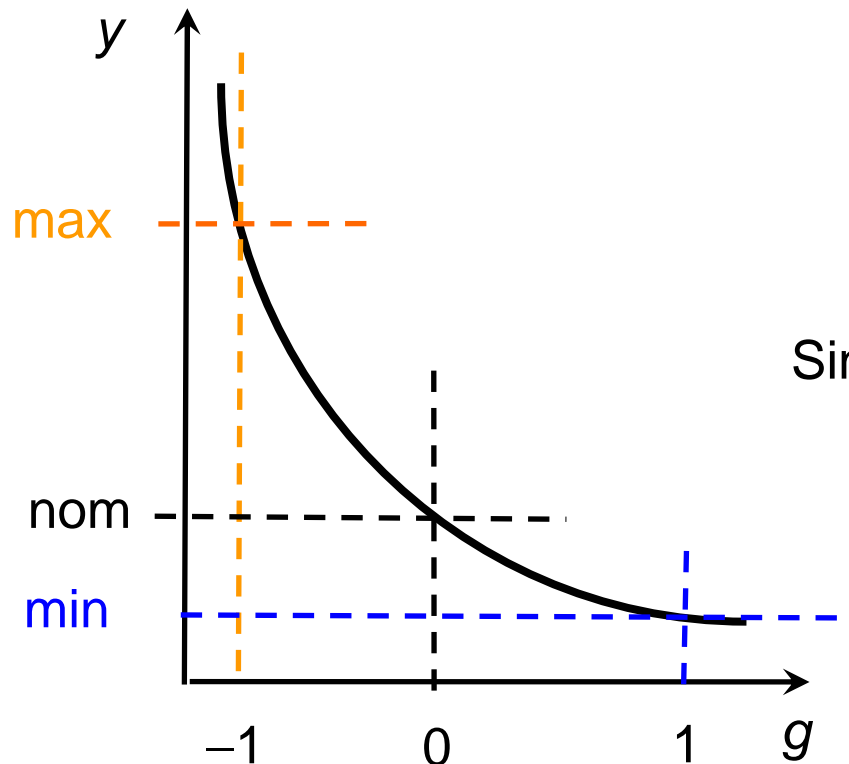


Element 2. Our Solution (1)

A simple mapping relation:

$$y(g; b, n, w) = n + C \left(\frac{1}{1 + \alpha g} - 1 \right)$$

Always monotonic



n – nominal value

b – 3σ best-case value

(could be either max or min)

w – 3σ worst-case value

(could be either min or max)

g – a symmetric distribution

C, α – Two coefficients

Singularity at $(1 + \alpha g) = 0$ provides the capability of modeling a very asymmetric distribution

Element 2: Our Solution (2)

Its solution:

$$\text{asym}(g; b, n, w) = n + \frac{2(w-n)(b-n)g}{w-b + (w+b-2n)g}$$

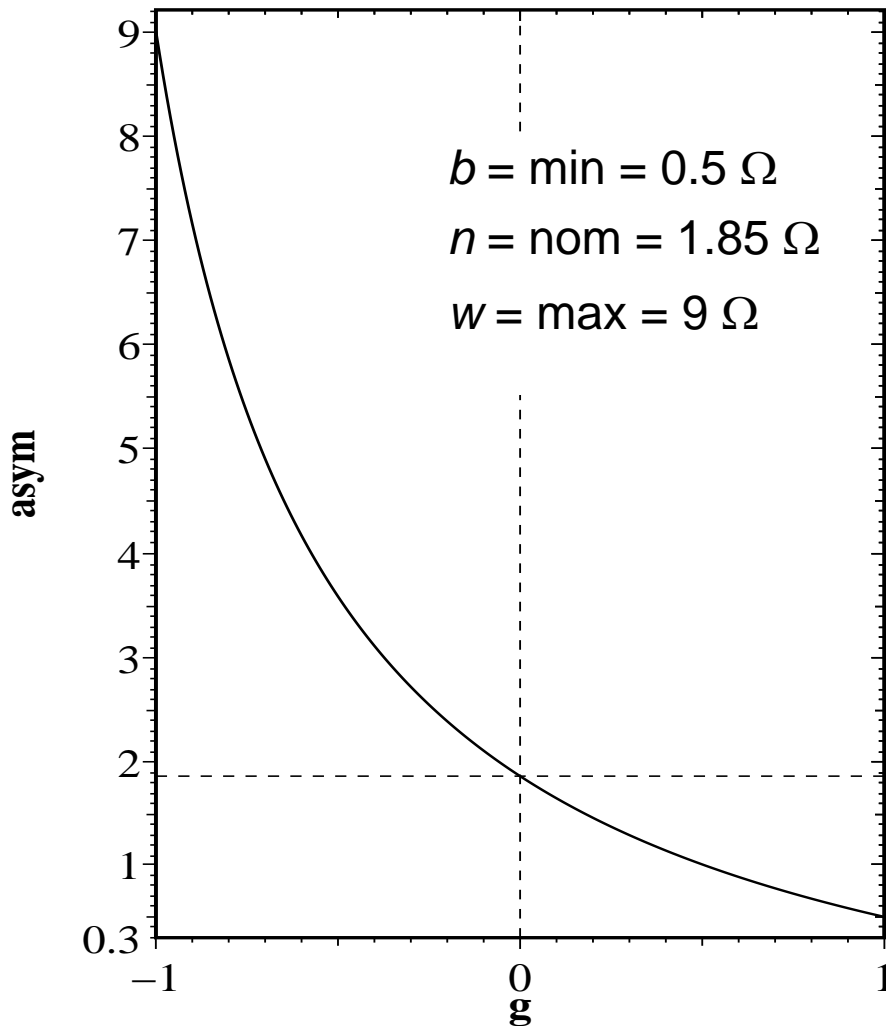
is simple and powerful

**When a distribution is symmetric,
the above solution simplifies to:**

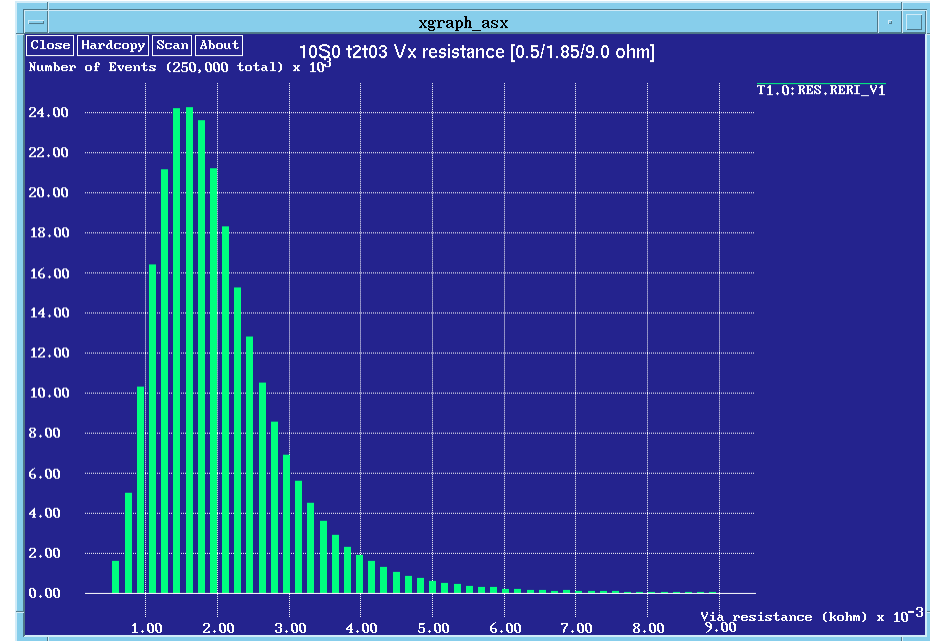
$$\text{sym}(g; b, n, w) = n + (b-n)g, \quad w+b = 2n$$

Element 2: Via resistance example

Mapping relation



Histogram of MC simulation



Element 2: Other Mapping Relations

There are other forms of mapping relations to model asymmetric distributions

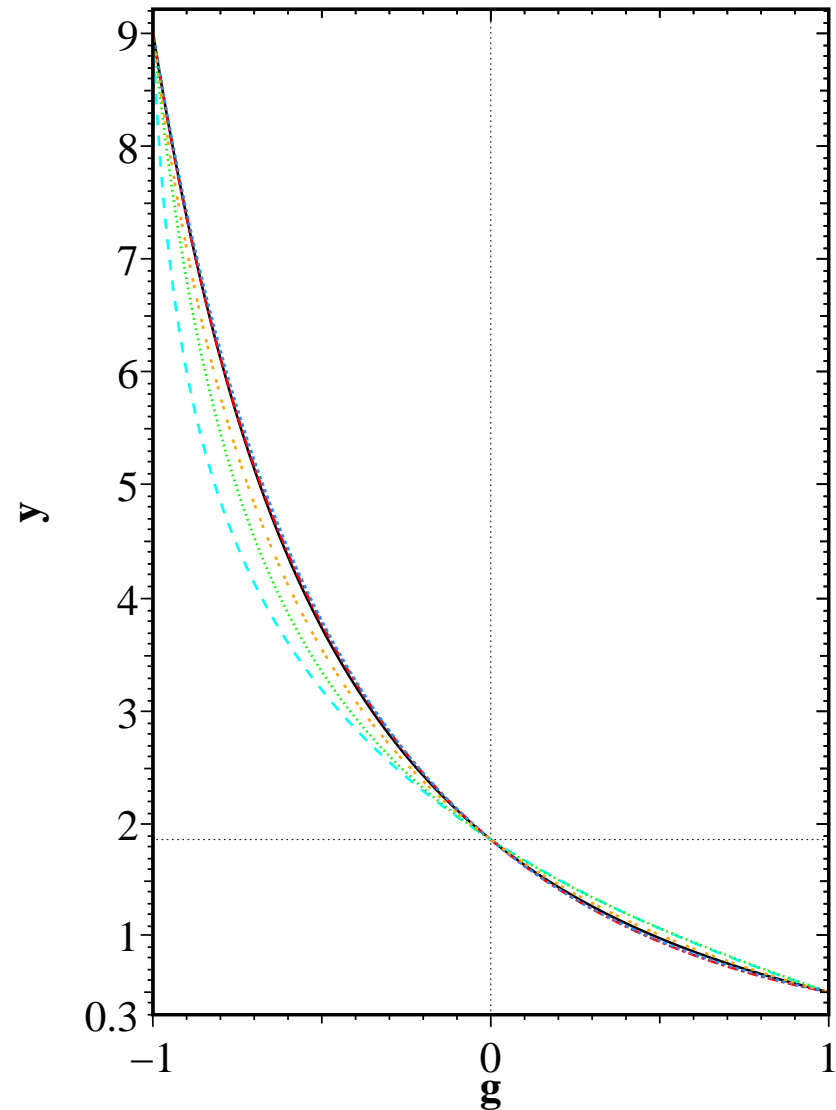
For the same via resistance example:

$$b = \min = 0.5 \Omega$$

$$n = \text{nom} = 1.85 \Omega$$

$$w = \max = 9 \Omega$$

Explicit expressions are given in the paper



3. Treatment of Many Statistical Distributions in Corner Simulations

- **Two opposing requirements:**
 - **A. For ease of use, we prefer to have only a few skewing parameters to control many process distributions. e.g.,**
 - `cor_res` for poly & diffusion resistors
 - `cor_cap` for junction & MIM capacitors
 - `cor_wirerc` for interconnect parasitic R and C
 - **B. To find all sensitivities of a performance target to all process distributions, to reach an optimal corner, or to find a common corner of 2 or more performance targets, we also prefer to have an independent skewing parameter for each independent process distribution**

3. Our Solution: A hierarchical structure for many skewing parameters

Higher-level skewing parameter

Process distributions

Lower-level skewing parameters

N+ diffusion resistor's sheet resistance

ndrs_sigma

N+ diffusion resistor's end resistance

ndre_sigma

...

P+ poly resistor's sheet resistance

ppcrs_sigma

P+ poly resistor's sheet resistance

ppcre_sigma

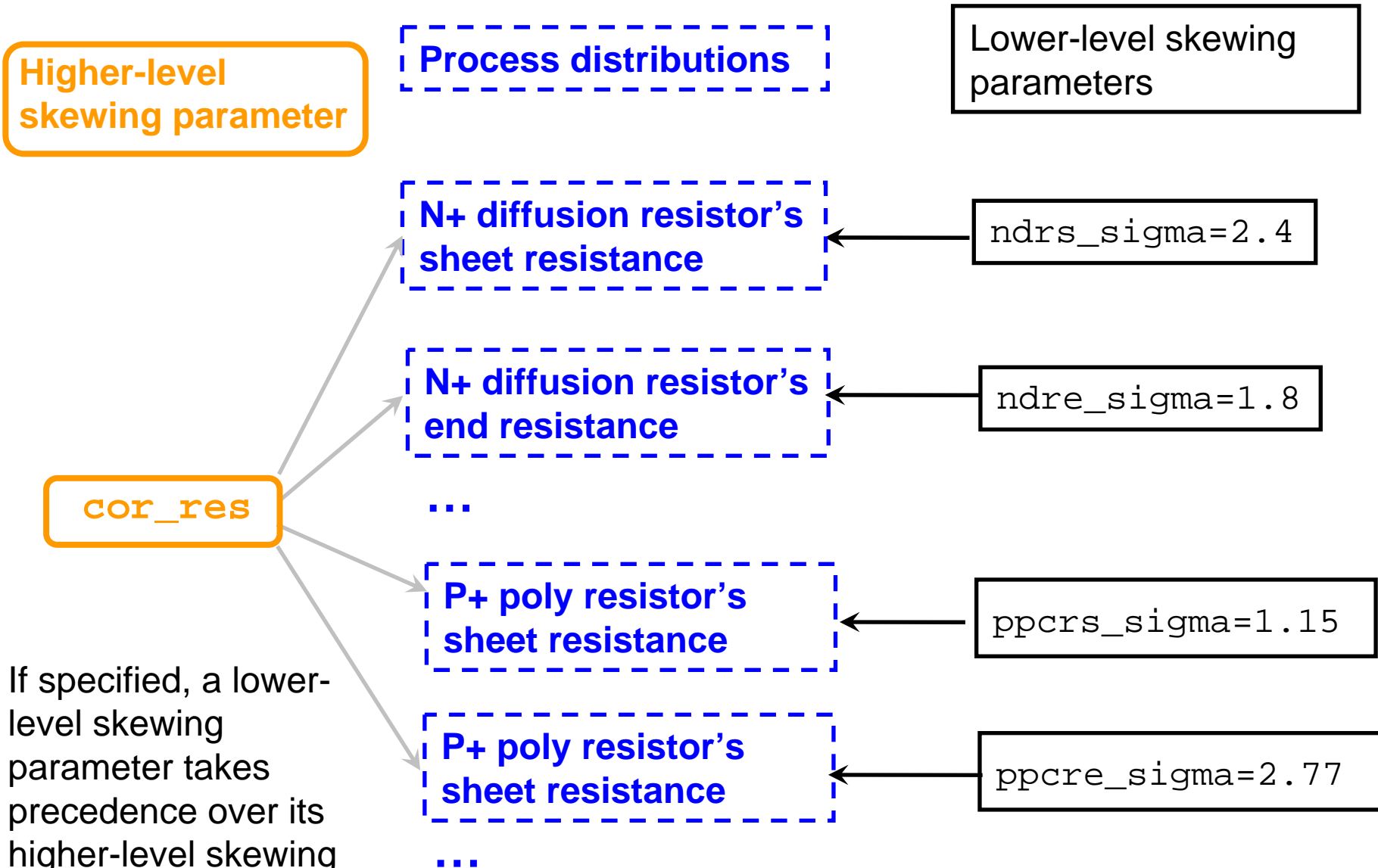
...

Typical range is [-3, +3]

cor_res

It controls the skewing of multiple process distributions at default

3. Our Solution: A hierarchical structure (2)



If specified, a lower-level skewing parameter takes precedence over its higher-level skewing parameter

4. Modeling the Combined Effect of Chip-mean and Across-chip Variations

- **A. Combining chip-mean (CM) variation and across-chip variation (ACV) for a single model parameter**
 - **When ACV is treated as completely uncorrelated (e.g., V_{th0})**

$$p_i = n + \sigma_{cm}G + \sigma_{acv}g_i, \quad i = 1, 2, 3, \dots,$$

- **When ACV is treated as completely correlated (e.g., channel length and width)**

$$p_i = n + \sigma_{cm}G + \sigma_{acv}g_0, \quad i = 1, 2, 3, \dots.$$

$G, g_0, g_1, g_2, g_3, \dots$ – stochastic/random variables

Total variation of p :
$$\sigma_p = \sqrt{\sigma_{cm}^2 + \sigma_{acv}^2}$$

4. (Systematic + Random) Variations

- B. When an ACV is characterized as a percentage of the mean (nominal) value:

$$\sigma_{acv} = n\sigma_{mm}$$

Typical treatment:

$$p_i = (n + \sigma_{cm}G)(1 + \sigma_{mm}g_i), \quad i = 1, 2, 3, \dots$$
$$= n + \sigma_{cm}G + n\sigma_{mm}g_i + \sigma_{cm}G\sigma_{mm}g_i$$

Nominal + **Chip-mean variation** + ACV + **Un-expected coupling term**

Correct treatment: $p_i = n + \sigma_{cm}G + n\sigma_{mm}g_i, \quad i = 1, 2, 3, \dots$

Element 4.

- **C. Enabling MC simulations of ACV at a chip-mean corner**
 - **Designers often need to ensure the functionality of their circuits at the worst chip-mean corner**

$$p_i = n + \sigma_{cm}s + \sigma_{acv}g_i, \quad i = 1, 2, 3, \dots$$

s – a skewing parameter

g_i – stochastic/random variables

Enabling circuit designers to performance a smaller number of MC runs at the chip-mean corner to obtain a larger sample of ACV (or mismatch).

Summary

- Reviewed requirements for statistical SPICE models
- Presented our solutions to several elements in statistical SPICE models
- Our solutions establish a solid foundation for a good statistical SPICE models
- Many of our solutions have been used in IBM bulk Common Platform's and SOI Alliance's compact models

**Thank S. Khandelwal (at IBM India) for
presenting this talk for us at 2009
Nanotech/WCM**