

# Theory and Modeling Techniques Used in PSP Model

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***G. Gildenblat, X. Li, H. Wang, W. Wu and A. Jha***

Department of Electrical Engineering  
The Pennsylvania State University, USA

and

***R. van Langevelde, A.J. Scholten, G.D.J. Smit and D.B.M. Klaassen***

Philips Research Laboratories, The Netherlands

# OUTLINE

- ❑ PSP Project Overview
- ❑ Surface Potential
- ❑ Retrograde Doping
- ❑ Symmetric Linearization
- ❑ Comparison with Pao-Sah Model
- ❑ Mobility Model
- ❑ Symmetry
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- ❑ Conclusions

# PSP Project Overview

- ❑ Development of the most advanced surface-potential-based model by merging SP and MM11
- ❑ Increasing physical content of the standard MOSFET model by solving several long-standing problems of compact modeling
- ❑ Incorporating all or most of the features required and expected by model user community for the next generation of compact models
- ❑ Working in close contact with the industry as represented by Compact Model Council
- ❑ Whenever possible, incorporating the best features of other compact models

# Surface Potential

## Considerations

- ❑ Surface potential equations (SPE)
- ❑ Accuracy of analytical approximation including large forward biases of s/b junction
- ❑ Retrograde profiles

# Different Forms of SPE

$$(V_{\text{GB}} - V_{\text{FB}} - \psi_s)^2 = \gamma^2 \phi_T \left\{ e^{-u} + u - 1 + \frac{n_b}{p_b} k_n [e^u - u - 1 - \chi(u)] \right\}$$

$$u = \psi_s / \phi_T; \quad k_n = \exp(-V_n / \phi_T)$$

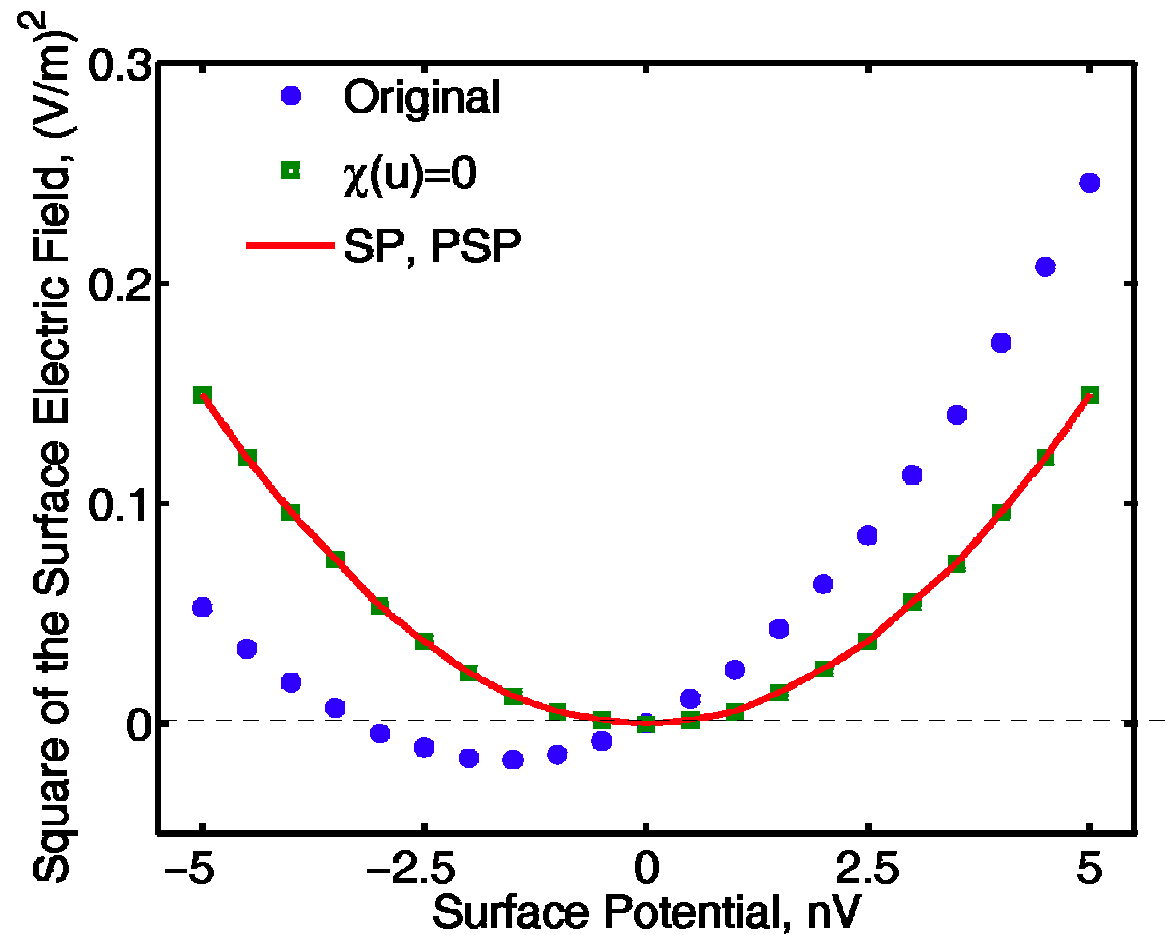
Pao-Sah:  $\chi(u) = u(k_n^{-1} - 1)$

McAndrew and Victory, Wu et al.:  $\chi(u) = 0$

PSP:  $\chi(u) = u^2 / (2 + u^2)$

- ❑ All forms of SPE are numerically identical except for
  - narrow regions (nV) near flat-band voltage
  - high forward bias
  
- ❑ This has nothing to do with numerical or analytical calculation of surface potential

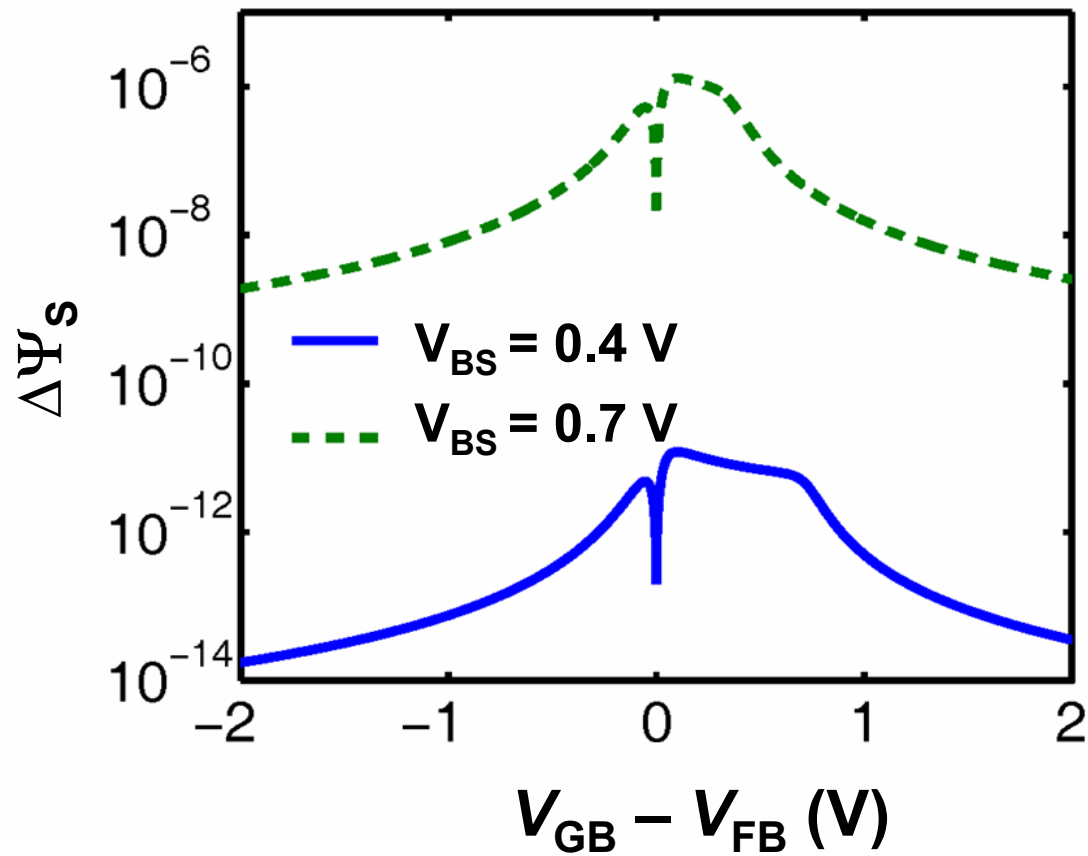
# Bug that Escaped Detection for 35 years



C. McAndrew and J. Victory, 2002;

W. Wu, T.L. Chen, G. Gildenblat and C. McAndrew, 2004

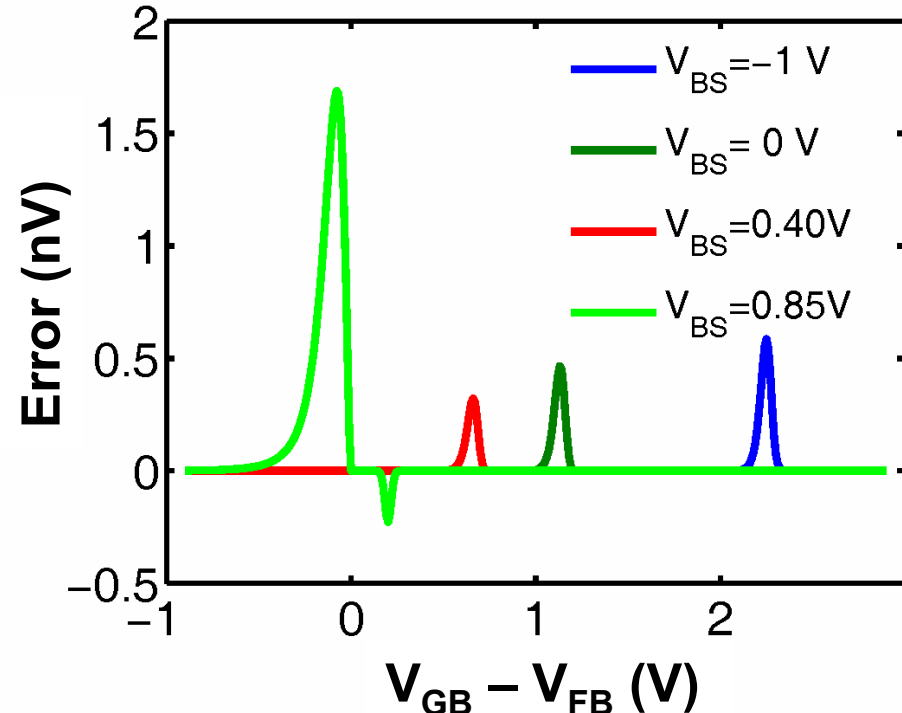
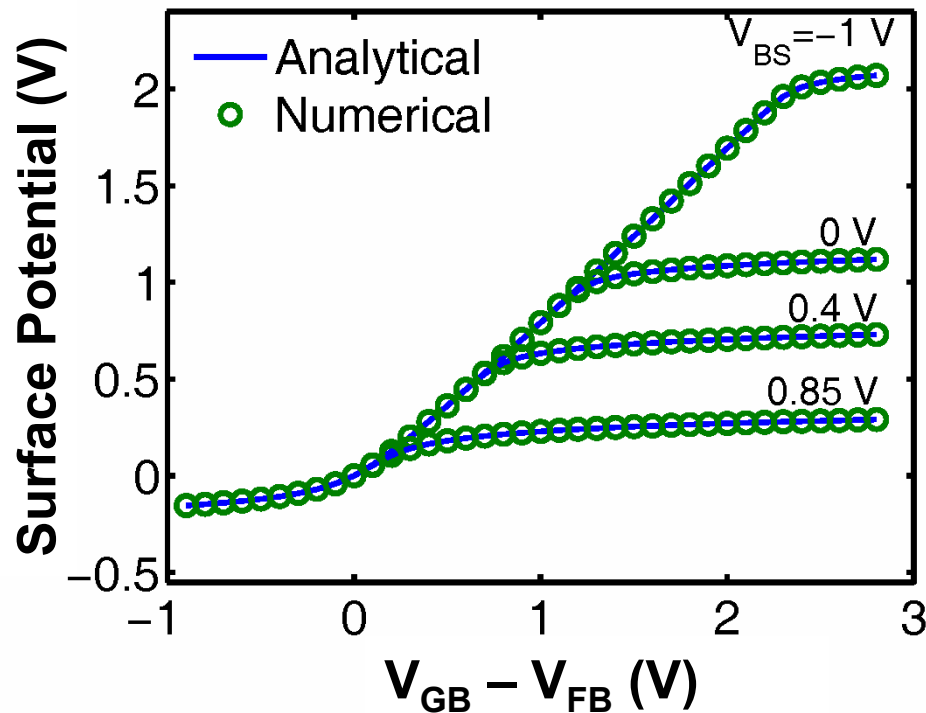
# Surface Potential Difference for $\chi(u)=0$ and $\chi(u)=u^2/(2+u^2)$



For  $V_{BS} \leq 0$  the difference is below 1 fV.

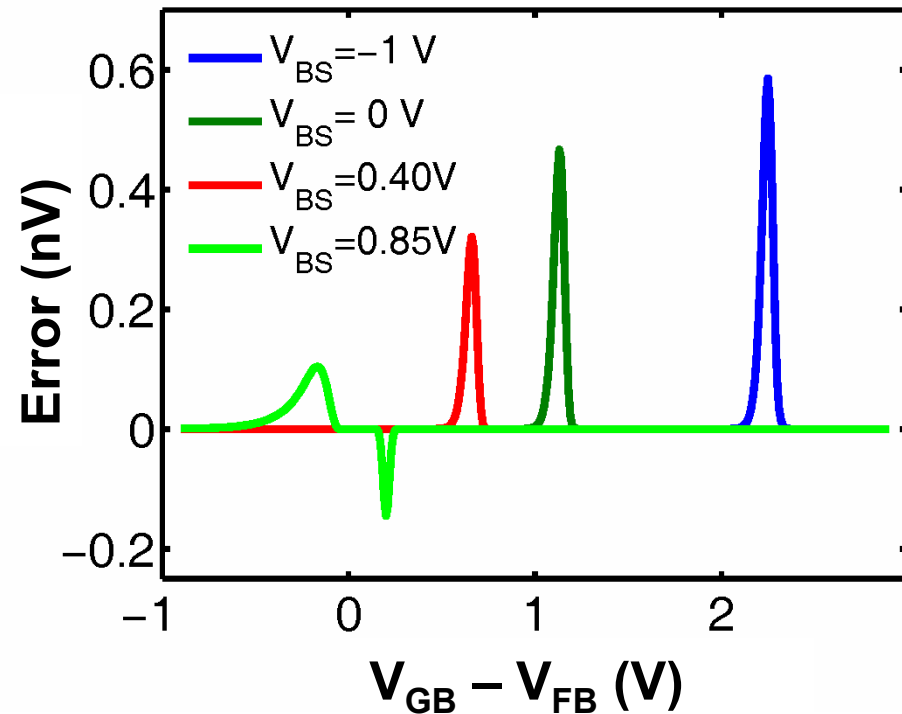
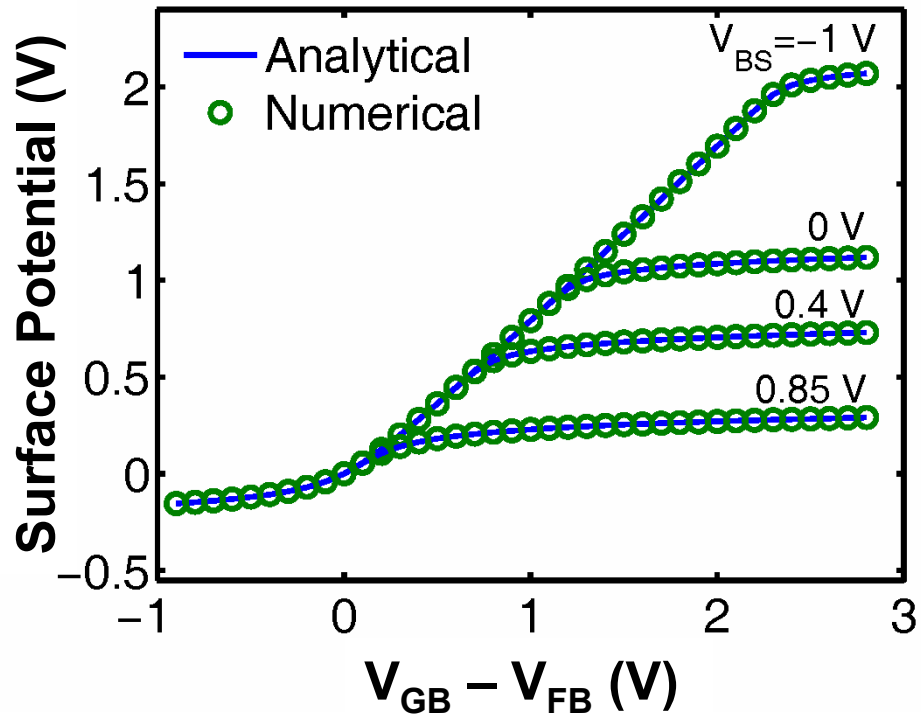
$N_{SUB} = 5 \times 10^{17} \text{cm}^{-3}$ ,  $T_{ox} = 2 \text{nm}$ ,  $V_{FB} = -1 \text{V}$ .

# Analytical Approximation for Surface Potential, $\chi(u)=0$



$$N_{SUB} = 5 \times 10^{17} \text{cm}^{-3}, T_{ox} = 2 \text{nm}, V_{FB} = -1 \text{V}.$$

# Analytical Approximation for Surface Potential, $\chi(u)=u^2/(2+u^2)$



$$N_{SUB} = 5 \times 10^{17} \text{cm}^{-3}, T_{ox} = 2 \text{nm}, V_{FB} = -1 \text{V}.$$

# SPE Summary

- ❑ Rigorous integration of Boltzmann-Poisson Equation is possible for MOSCAP, impossible for MOSFET (imref splitting is position dependent)
- ❑ Pao-Sah approximation of SPE is excellent except within 1-2 nV of flat-band
- ❑ Modifications (conditioning) of Pao-Sah SPE are neither less nor more rigorous than original formulation
- ❑ Outside of extremely narrow region near flat-band different forms of SPE are numerically indistinguishable

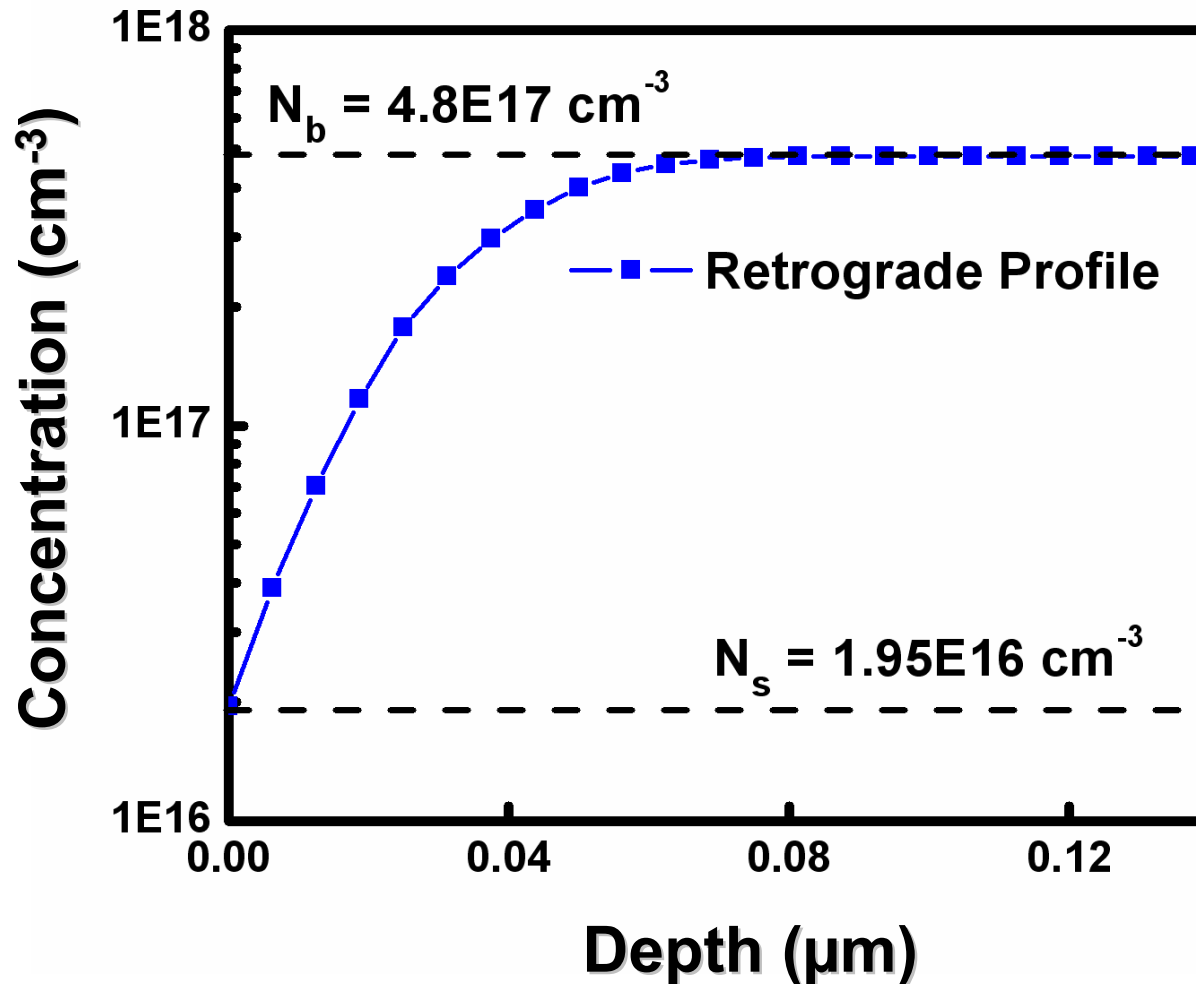
# SPE Summary (continued)

- ❑ Whatever form of SPE is selected, analytical approximation is perfectly accurate
- ❑ Evaluate the accuracy using the same form of SPE for numerical and analytical solutions
- ❑ No negative consequences of any kind from using analytical approximation

# Retrograde Doping

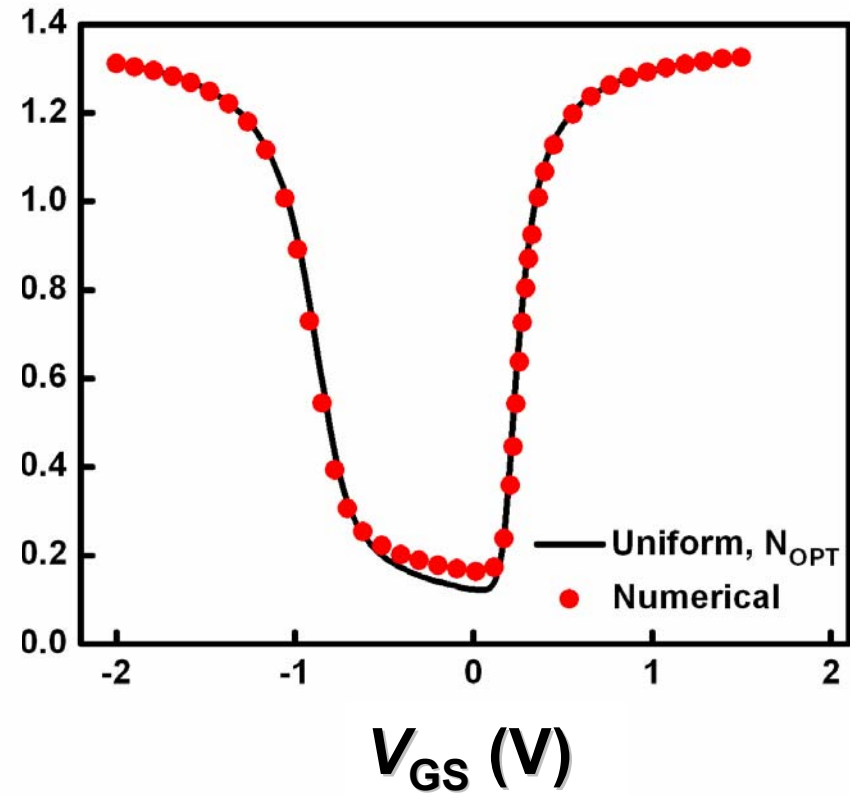
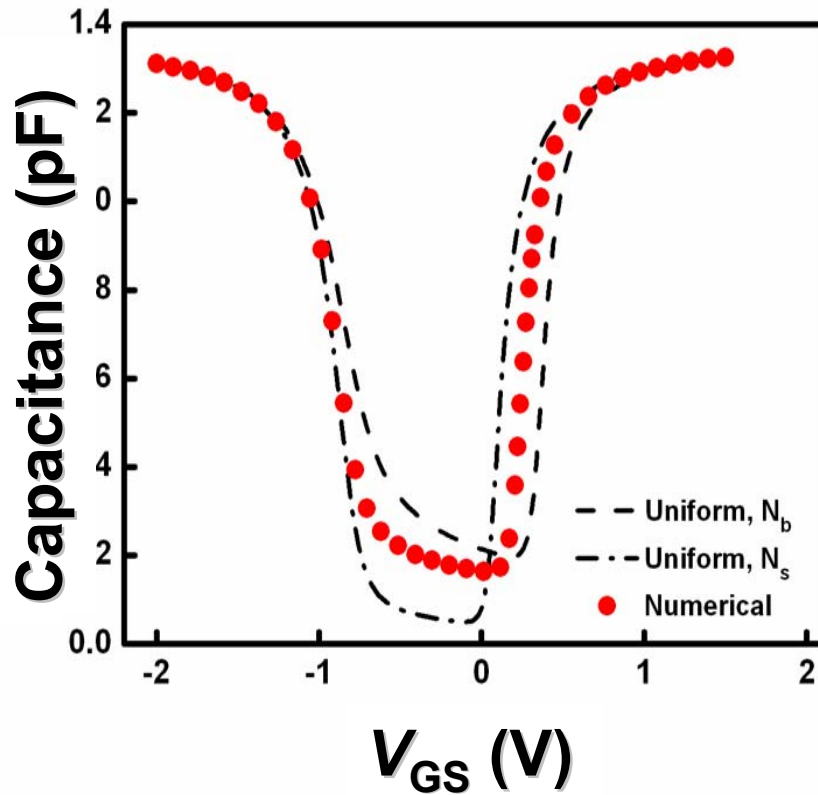
- ❑ Device characteristics are not as drastically affected as one may think
- ❑ Research in this direction has just begun
- ❑ Initial results are promising
- ❑ One idea: incremental charge is controlled by the doping at the depletion layer edge, effective doping is bias dependent

# Retrograde Doping Profile



From P. Packan, Intel. Presented with permission.

# $C_{GG}-V_{GS}$ Plots for Different Substrate Doping Levels



$N_b = 4.8 \times 10^{17} \text{ cm}^{-3}$ ,  $N_s = 1.95 \times 10^{17} \text{ cm}^{-3}$ , and  $N_{OPT} = 1.5 \times 10^{17} \text{ cm}^{-3}$

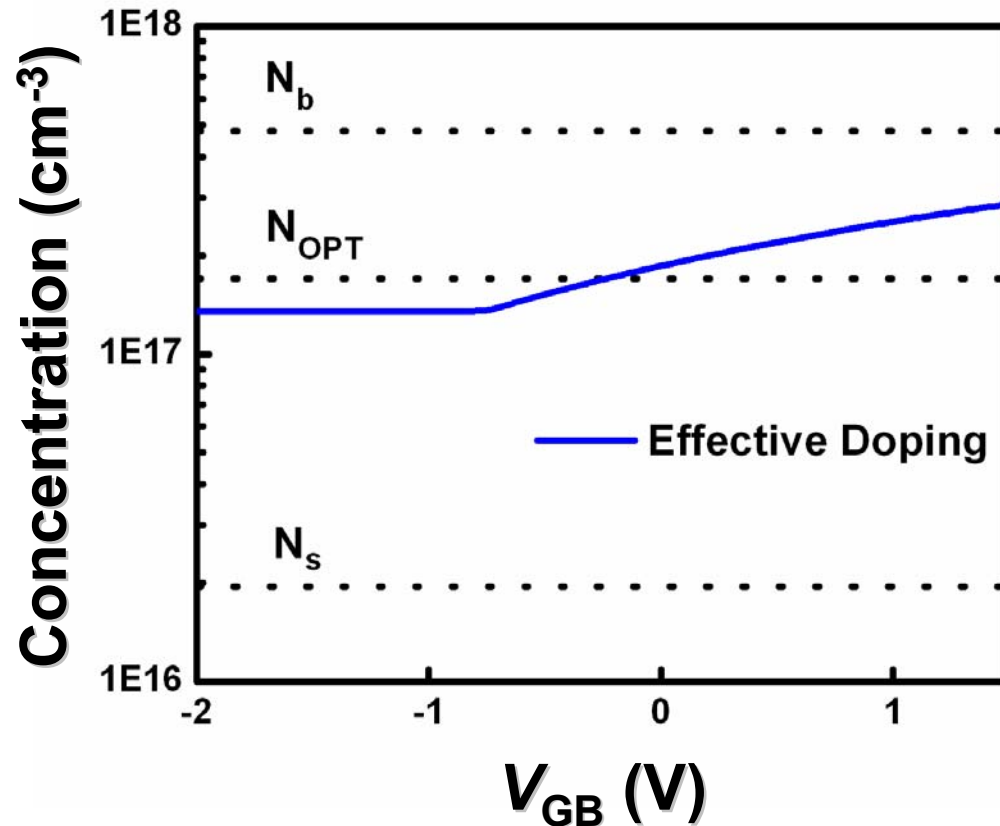
# Bias-Dependent Doping

$$(V_{\text{GB}} - V_{\text{FB}} - \psi_s)^2 = \gamma^2 \phi_T \left\{ e^{-u} + u - 1 + \frac{n_b}{p_b} k_n [e^u - u - 1 - \chi(u)] \right\}$$

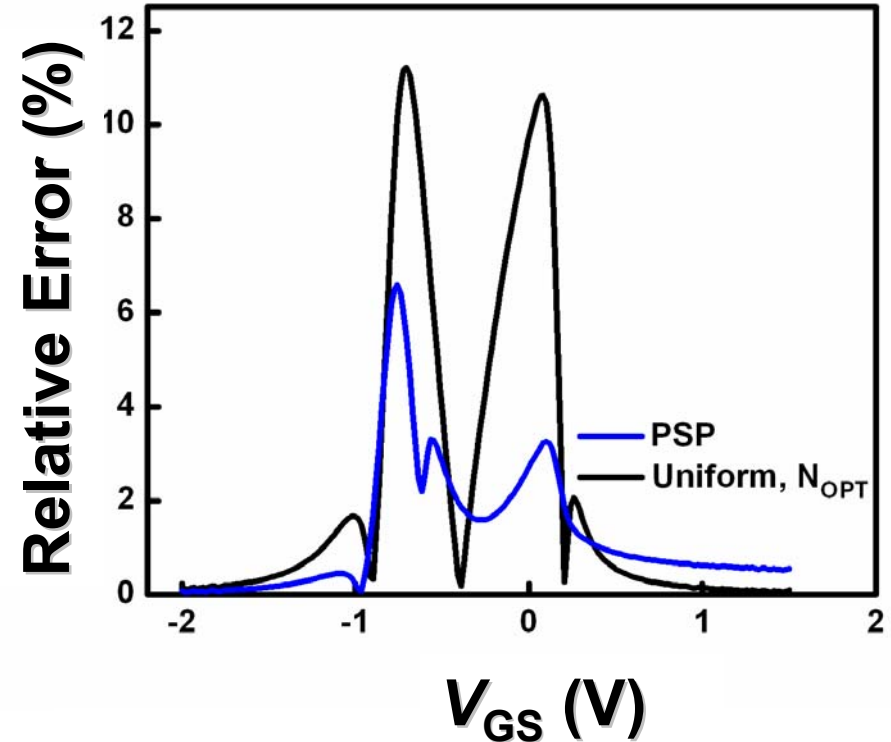
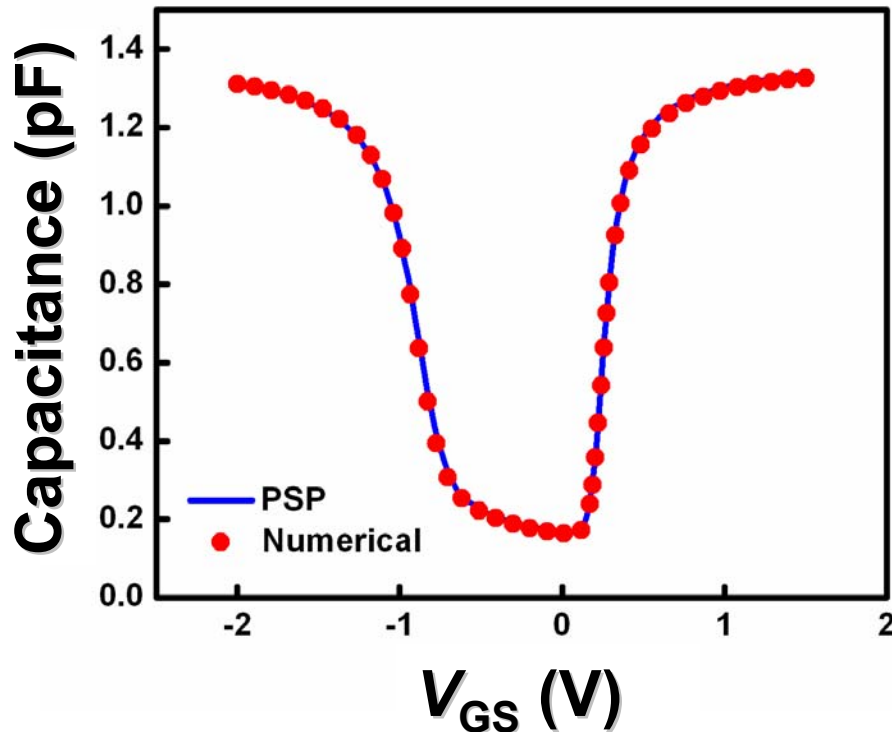
- Arora, 1987:  $\gamma = \gamma(\psi_s)$ 
  - One-step approximation
  - Cannot easily be used in  $\psi_s$ - based models
  - SPE functional form changes
  
- PSP:  $\gamma = \gamma(V_{\text{GB}})$ 
  - Same physics, different variable
  - SPE structure remains unchanged
  - Nothing else needs to be modified in the model

# Gate Bias Dependence of Effective Doping

$$N = N_{\text{SUB}} \left[ 1 + D_{\text{NSUB}} \cdot f_s(V_g - V_{\text{NSUB}}) \right] \quad f_s(v) = \frac{1}{2} \left( v + \sqrt{v^2 + \text{NSLP}} \right)$$



# Retrograde Doping in PSP



$NSUB = 1.35 \times 10^{17} \text{ cm}^{-3} \text{ cm}^{-3}$ ,  $DNSUB = 0.5$ ,  $VNSUB = -0.75$ , and  $NSLP = 0.001$

# Symmetric Linearization

## □ Objective

- Simple expressions for drain current and terminal charges
- Perfect symmetry
- High accuracy
- No negatives, no new trade-offs

## □ Details

- Insensitivity to the form of the velocity-field relation
- Higher-order conductances and transconductances
- Gummel and McAndrew symmetry tests

# Symmetric Linearization and Velocity Saturation

$$q_i = q_{im} - \alpha(\psi_s - \psi_m); \quad \psi_m = (\psi_{ss} + \psi_{sd})/2$$

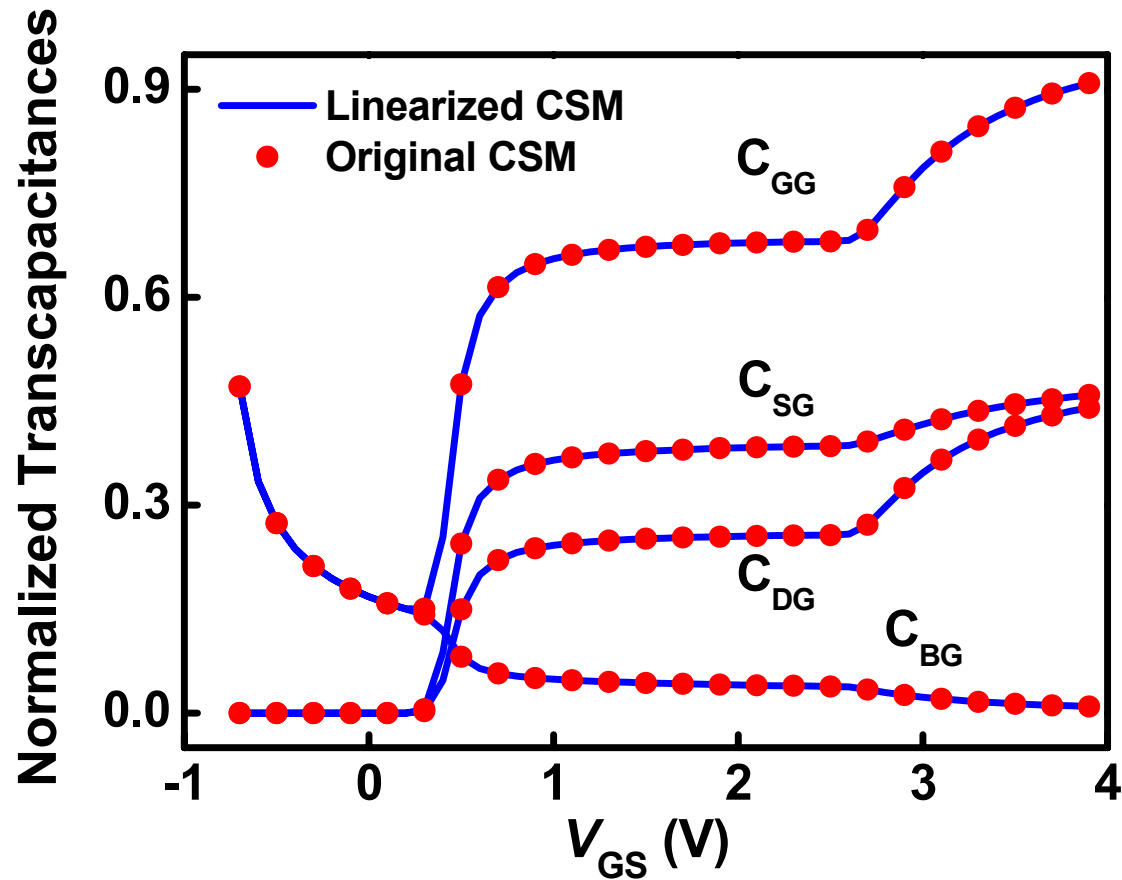
$$I_{DS} = \frac{\mu_{eff} W q_{im}^*}{\sqrt{1 + (E_y/E_c)^2}} \cdot \frac{d\psi_s}{dy}; \quad q_{im}^* = q_{im} + \alpha\phi_T$$

$$I_{DS} = \frac{\mu_{eff}}{G_{vsat}} \cdot \frac{W}{L} \cdot q_{im}^* \cdot \Delta\psi; \quad \Delta\psi = \psi_{sd} - \psi_{ss}$$

$$G_{vsat} = \frac{1}{2} \left[ 1 + \sqrt{1 + 2(\theta_{sat} \Delta\psi)^2} \right]; \quad \theta_{sat} = \frac{\mu_{eff}}{v_{sat} \cdot L}$$

- ❑ Simple expressions for charges, noise, gate current, etc.
- ❑ All benchmark tests are satisfied.

# Verification of Symmetric Linearization

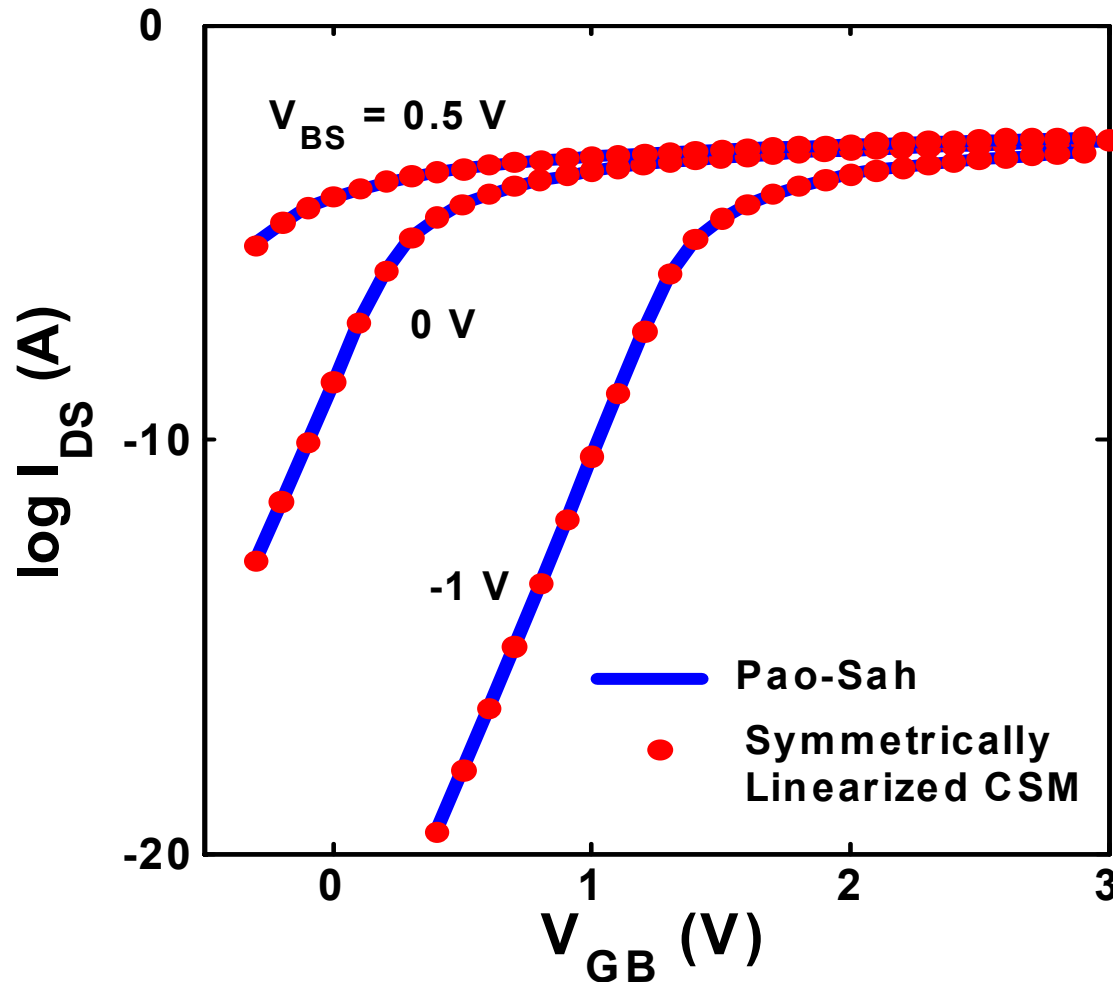


$$V_{DS} = 2V, V_{BS} = 0V, V_{FB} = -1V$$

# Comparison with Pao-Sah Model

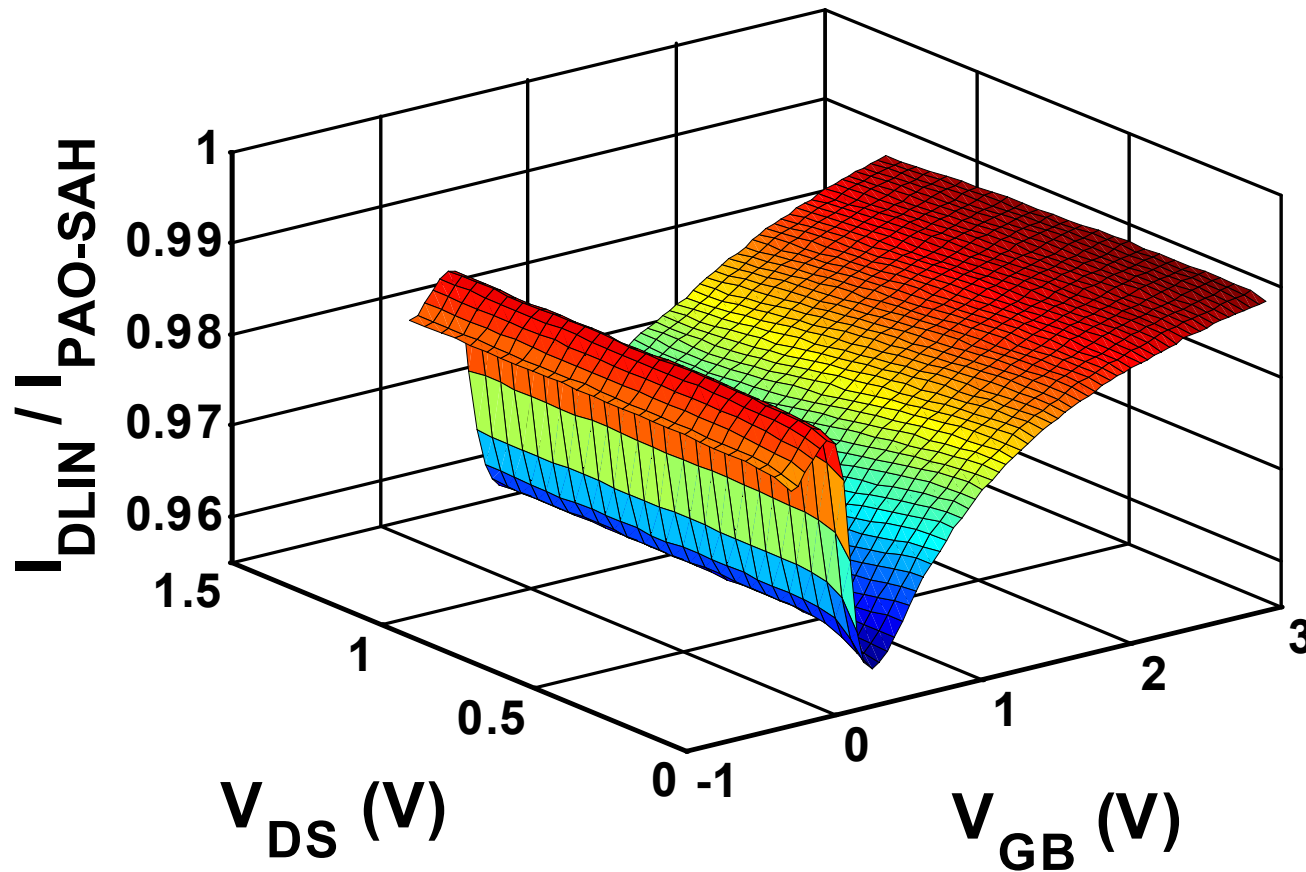
- ❑ Recent claims of 10% inaccuracy of CSM are inaccurate
- ❑ Direct comparison of symmetrically linearized CSM with Pao-Sah model validates symmetric linearization
- ❑ Accuracy is within 4% (usually better), totally adequate for compact modeling, other factors dominate

# Comparison of Symmetrically Linearized CSM and Pao-Sah Models



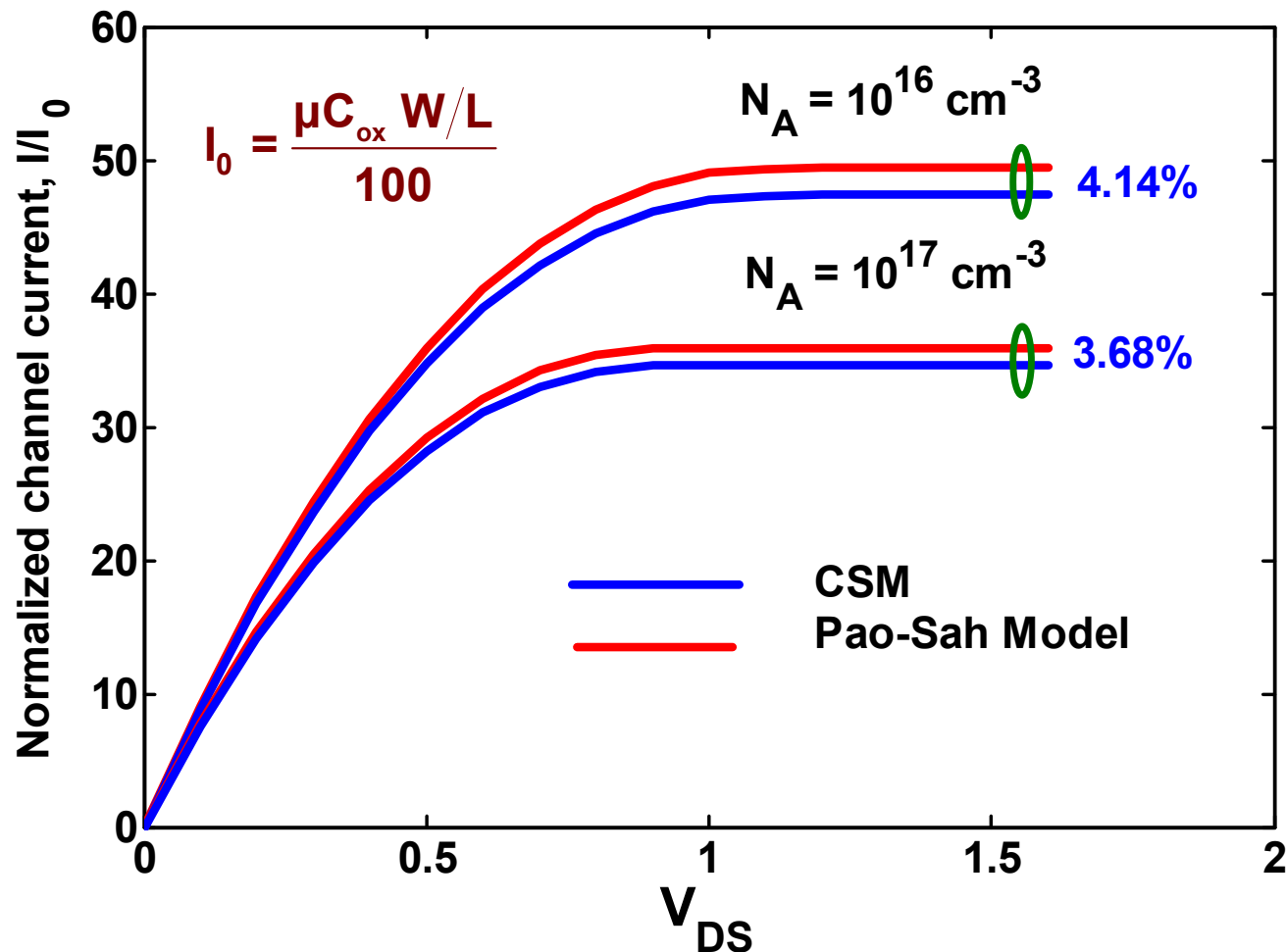
$T = 27^\circ\text{C}$ ,  $W/L = 2$ ,  $T_{\text{ox}} = 2$  nm,  $N_{\text{SUB}} = 5 \times 10^{17} \text{cm}^{-3}$ ,  $\mu = 400 \text{cm}^2/\text{Vs}$ ,  $V_{DS} = 0.5 \text{V}$ ,  $V_{FB} = -1 \text{V}$

# Comparison of Symmetrically Linearized CSM and Pao-Sah Models



$T = 27^{\circ}\text{C}$ ,  $W/L = 2$ ,  $T_{ox} = 2 \text{ nm}$ ,  $N_{SUB} = 5 \times 10^{17} \text{ cm}^{-3}$ ,  $\mu = 400 \text{ cm}^2/\text{Vs}$ ,  $V_{FB} = -1 \text{ V}$ ,  $V_{BS} = 0 \text{ V}$

# Comparison of Charge-Sheet and Pao-Sah Models (Revisited)



$T = 27^\circ\text{C}$ ,  $W/L = 10/10\mu\text{m}$ ,  $V_{FB} = -0.95 \text{ V}$ ,  $\mu = 500 \text{ cm}^2/\text{Vs}$ ,  $V_{GS} = 1\text{V}$ ,  $T_{OX} = 2\text{nm}$

# Recent Analysis of CSMs

**He et al. SSE, vol. 50, p. 263 (2006)**

- ❑ Accuracy of CSM relative to Pao-Sah is 10%.  
***Actually, 4% - see previous slide for the same parameter values as in the cited paper***
- ❑ Original SPE is “exact”  
***Not really, see above***
- ❑ SP, MM11, PSP do not include accumulation region in SPE  
***They do, see code, results***
- ❑ Symmetric linearization is “semi-empirical”  
***Reality: nothing empirical, no new parameters***  
***Incorrect analysis: differences between CSM and symmetrically linearized CSM are reported for  $V_{DS}=0$  (Fig. 4) where there is nothing to linearize.***

# Mobility Model

## □ User side

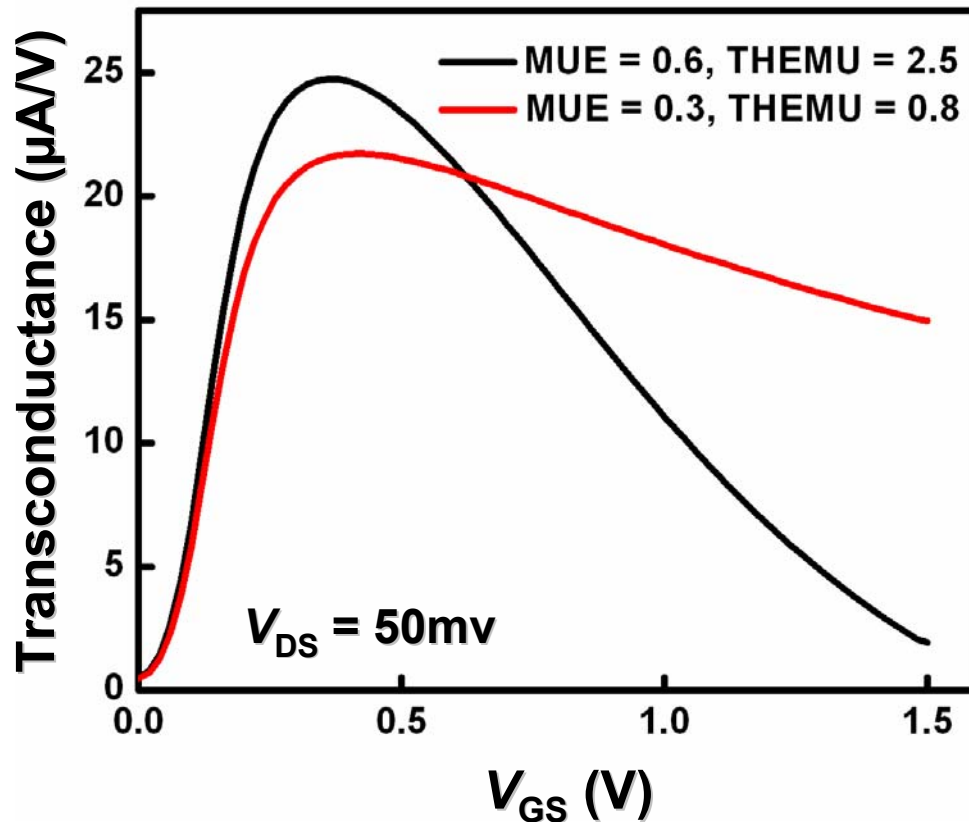
- Scalability
- All features that proved to be useful over the years
- Higher-order transconductances

## □ Physics

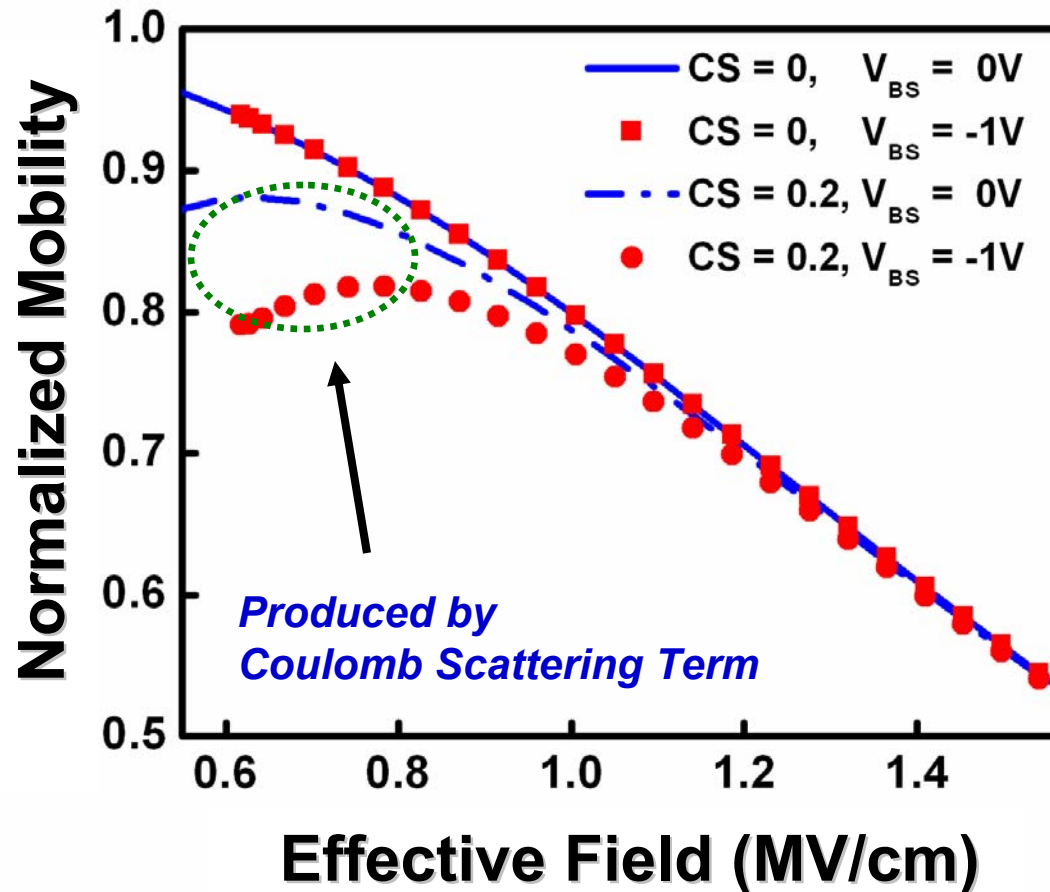
- Universal dependence on effective vertical field
- Deviations from universality, Coulomb scattering

# Mobility Model

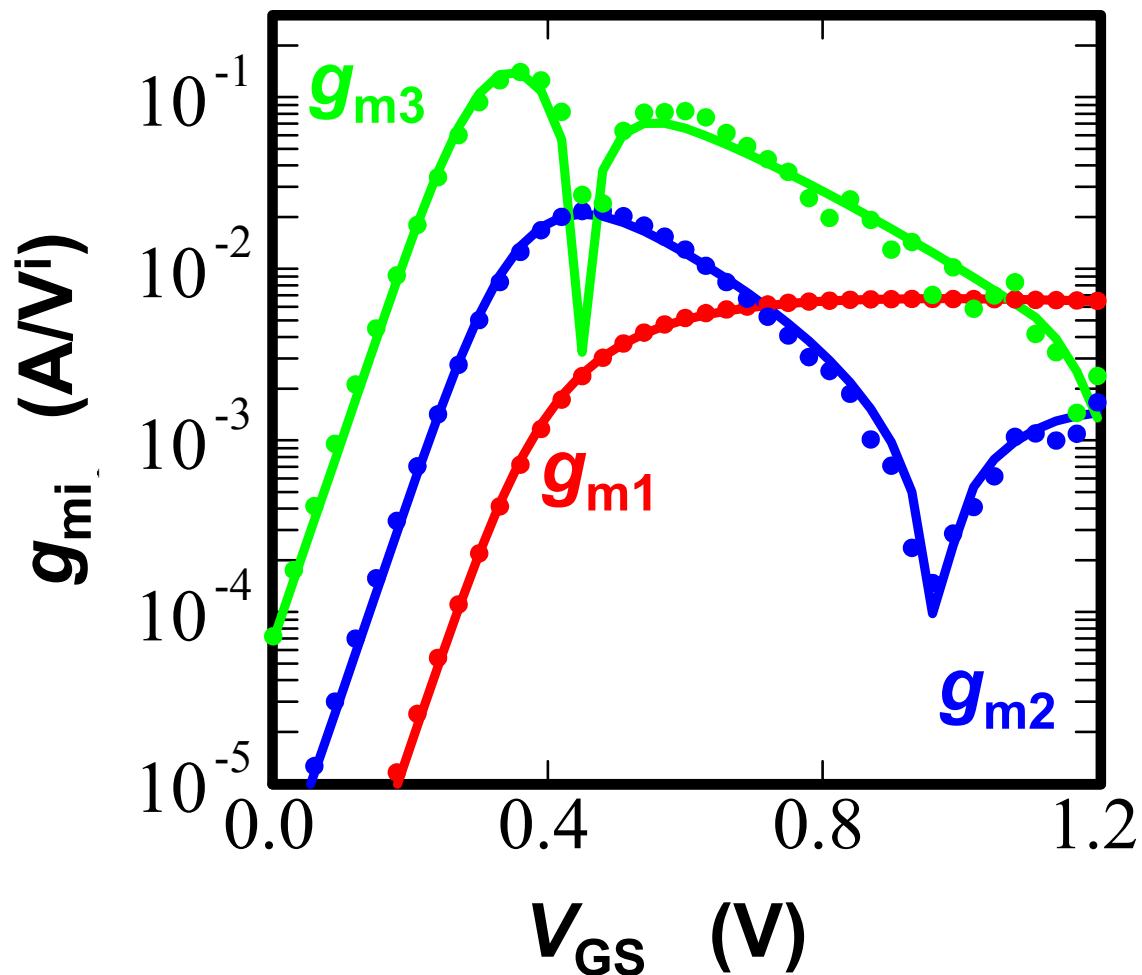
$$\mu_{\text{eff}} = \frac{M U_0 \cdot \mu_x}{1 + (M U E \cdot E_{\text{eff}})^{\text{THEMU}} + C S \cdot \left( \frac{q_{\text{bm}}}{q_{\text{bm}} + q_{\text{im}}} \right)^2 + G_R}$$



# Non-Universality of the Effective Mobility Produced by the Coulomb Scattering



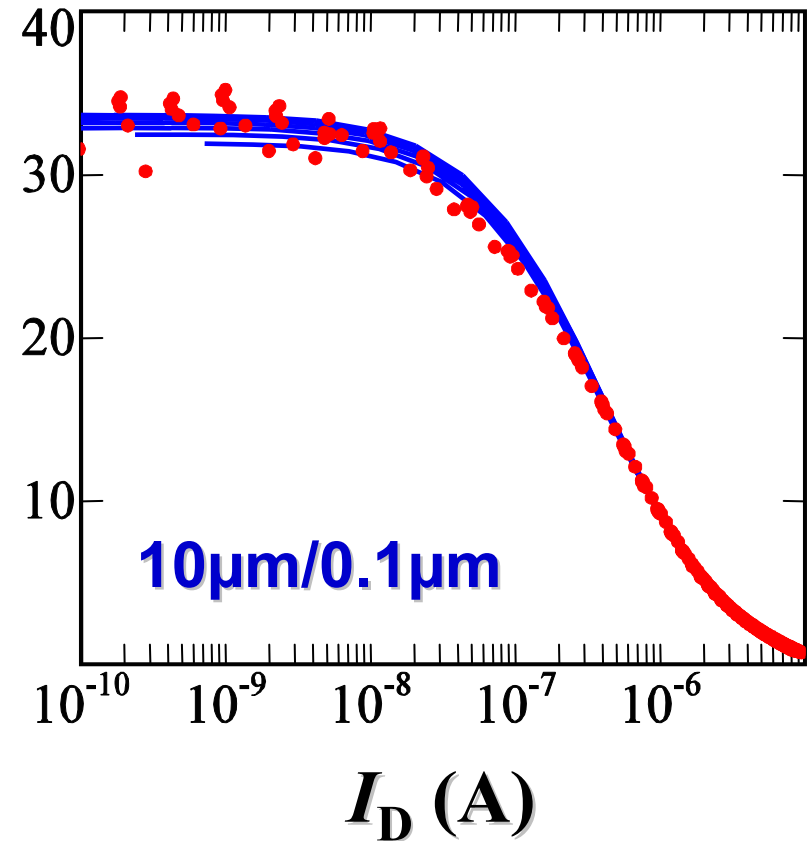
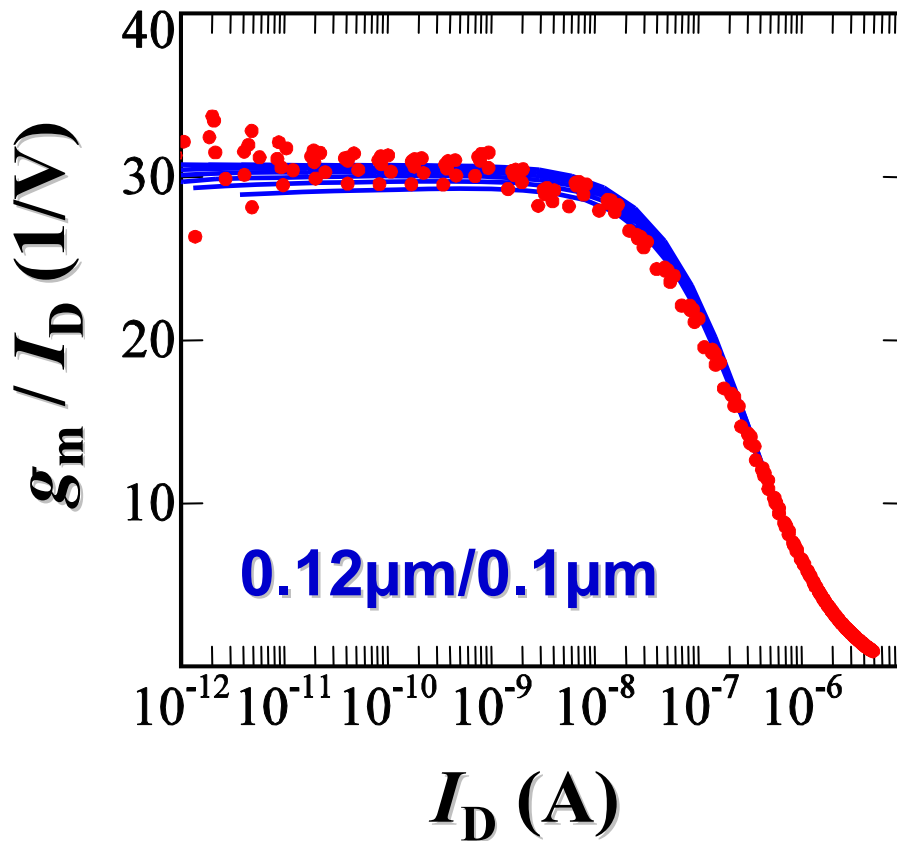
# Higher Order Transconductances



$$g_{mi} = \partial^i I_D / \partial V_{GS}^i$$

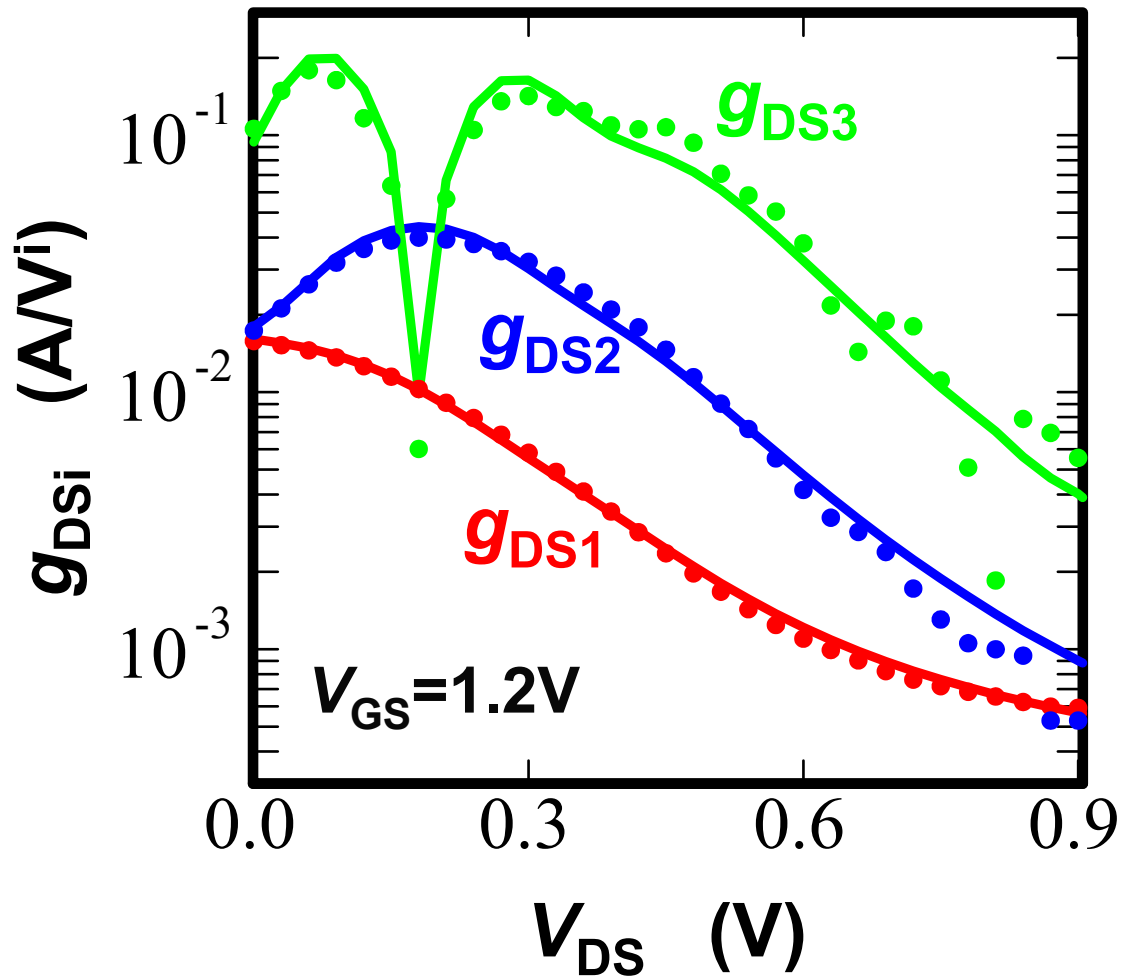
NMOS, W/L = 10  $\mu$ m/0.12  $\mu$ m,  $V_{DS}$  = 1.2 V,  $V_{BS}$  = 0 V

# $g_m/I_D$ for Two Corners of 90nm Process



$V_{DS} = 0.025\text{V}$ ,  $V_{SB} = 0$  to  $-1.2\text{V}$ , and  $V_{GS} = 0$  to  $1.2\text{V}$ .

# Higher Order Conductances

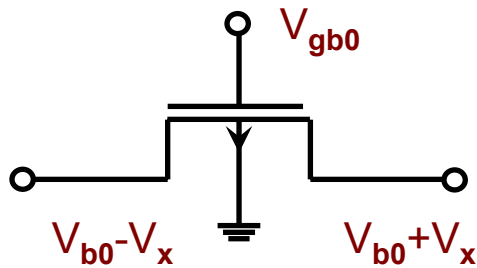


$$g_{mi} = \partial^i I_D / \partial V_{GS}^i$$

NMOS, W/L = 10  $\mu m$ /0.12  $\mu m$

# Symmetry

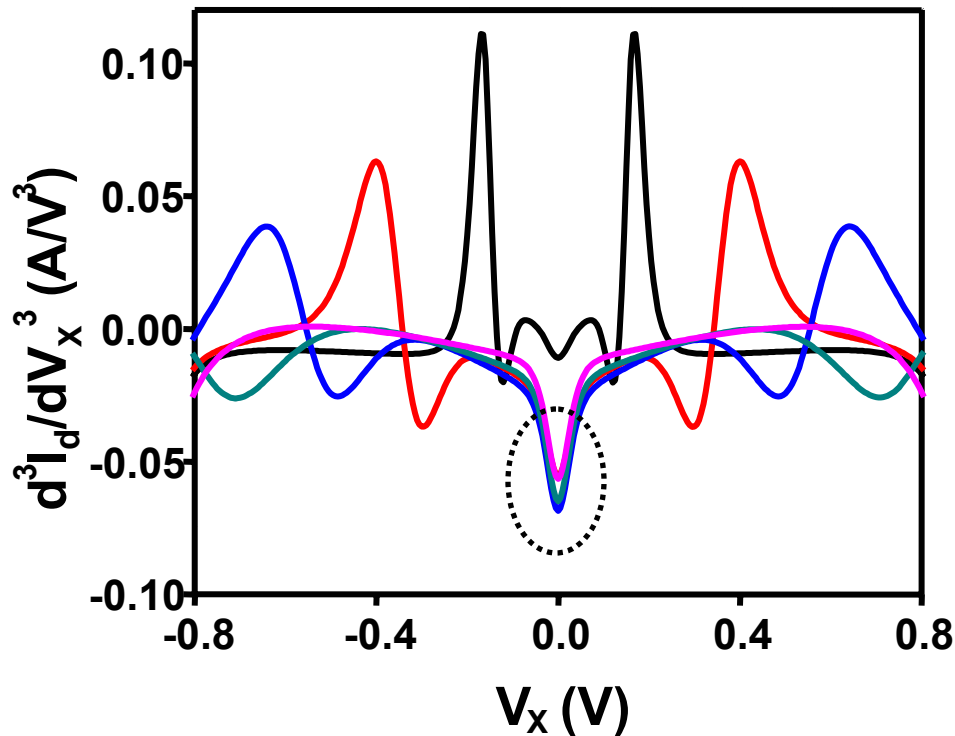
- ❑ Essential for RF applications
- ❑ New developments, better understanding
- ❑ Gummel Symmetry Test



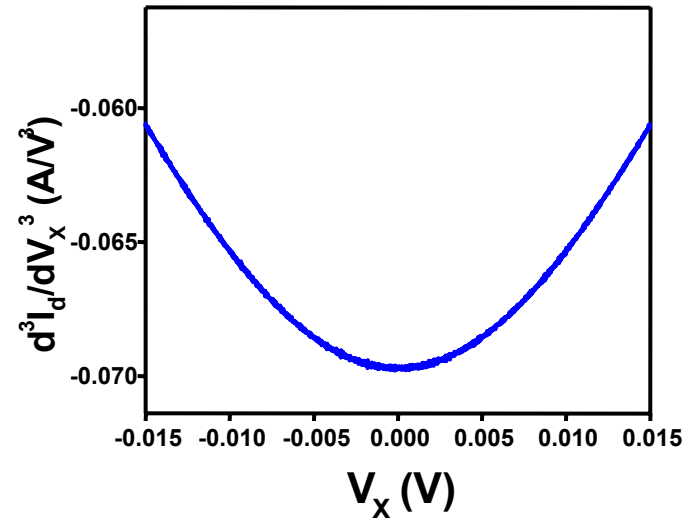
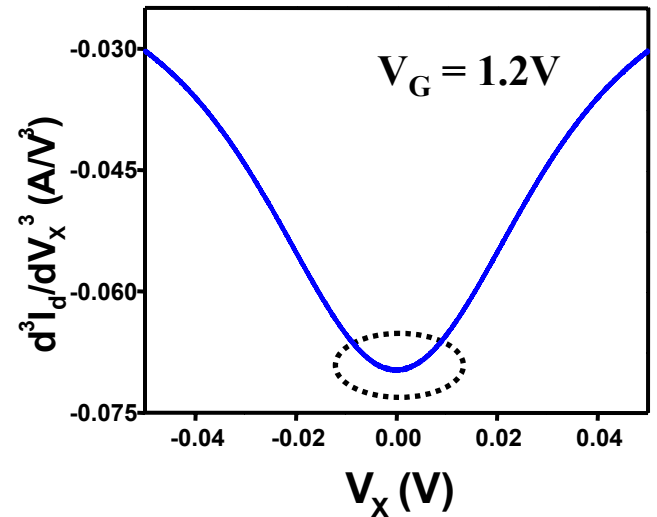
- ❑ Third derivatives are now routinely required by the industry
- ❑ McAndrew Symmetry Test

# Gummel Symmetry Test

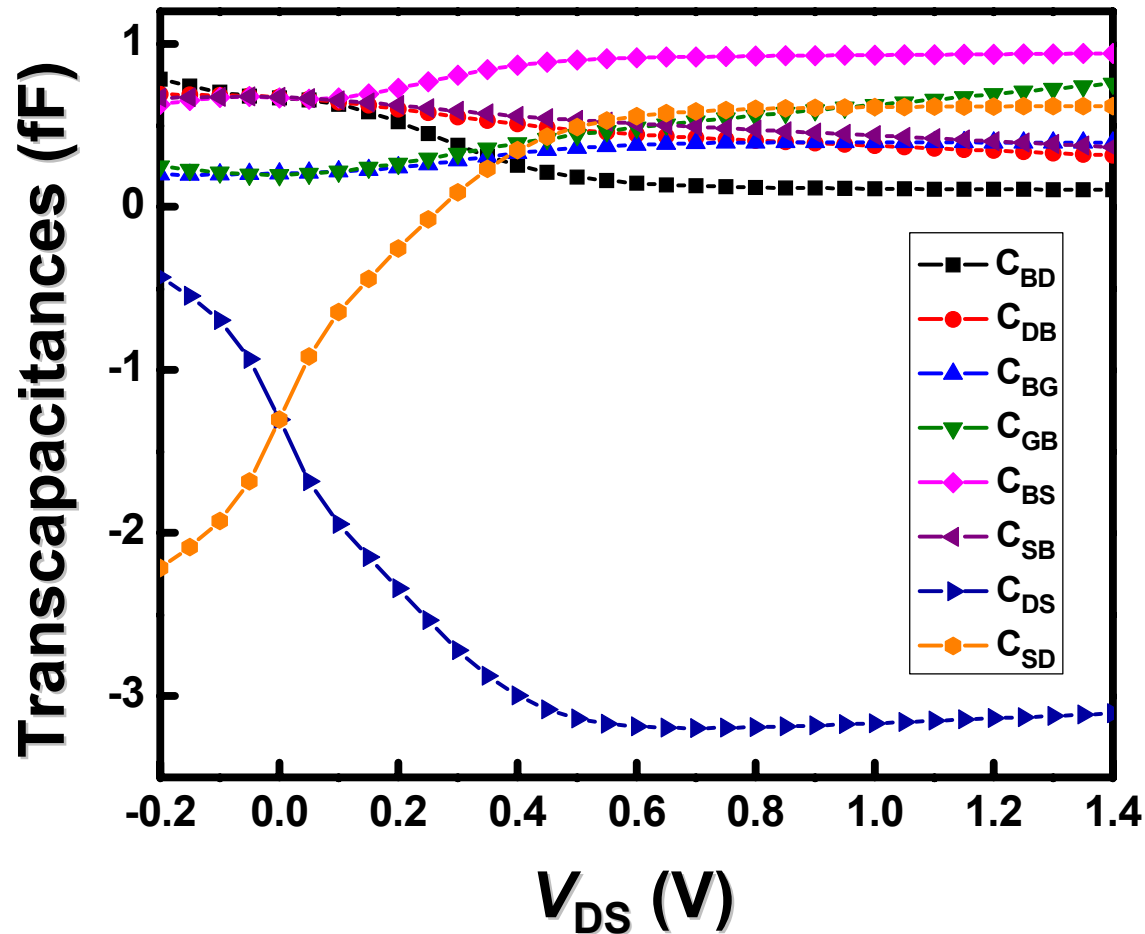
## 3<sup>rd</sup> Order Derivative of $I_d$



**PSP 101.0**

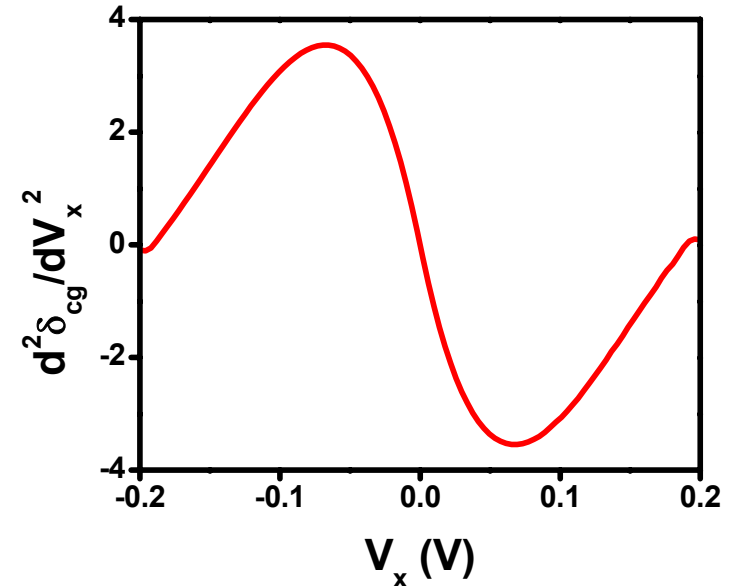
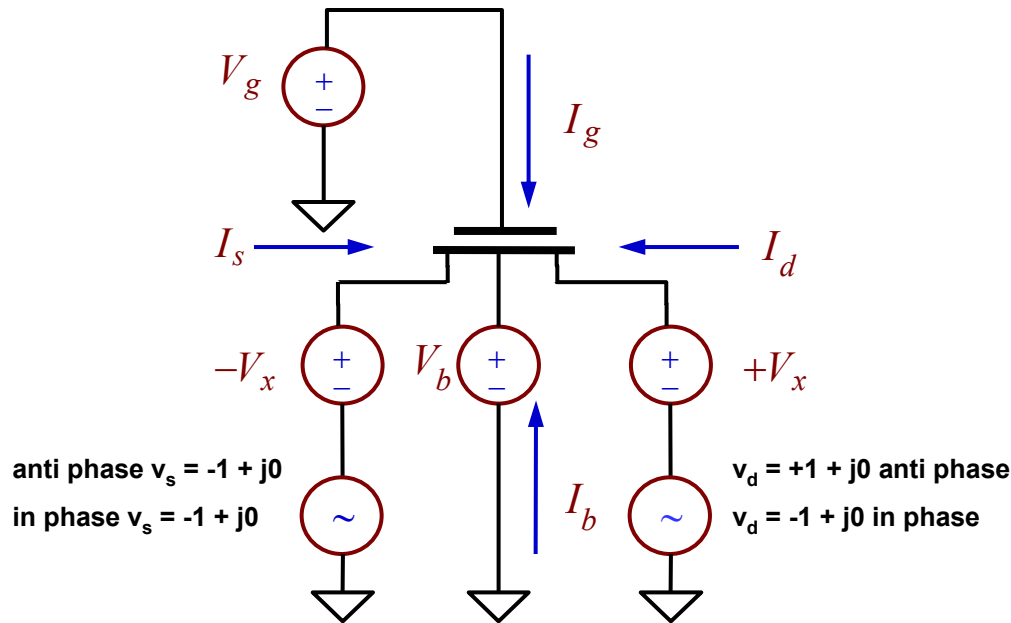


# Transcapacitances



$W/L=10/0.08\mu\text{m}$ ,  $V_{BS}=-0.1\text{V}$ ,  $V_{GS}=1.2\text{V}$

# McAndrew Symmetry Test AC version



$$\delta_{cg} = \frac{i_{g^-}}{i_{g^+}} = \frac{C_{gs} - C_{gd}}{C_{gs} + C_{gd}}; \quad \delta_{cb} = \frac{i_{b^-}}{i_{b^+}} = \frac{C_{bs} - C_{bd}}{C_{bs} + C_{bd}}$$

$$\delta_{csd} = \frac{(i_{s^-} + i_{d^-}) + (i_{s^+} - i_{d^+})}{(i_{s^-} - i_{d^-}) + (i_{s^+} + i_{d^+})} = \frac{C_{ss} - C_{dd}}{C_{ss} + C_{dd}}$$

**PSP 101.0**

W/L=10/0.08 $\mu$ m

$$i_{g^+} = \text{Im}(I_g [\text{in phase}])$$

$$i_{g^-} = \text{Im}(I_g [\text{anti phase}])$$

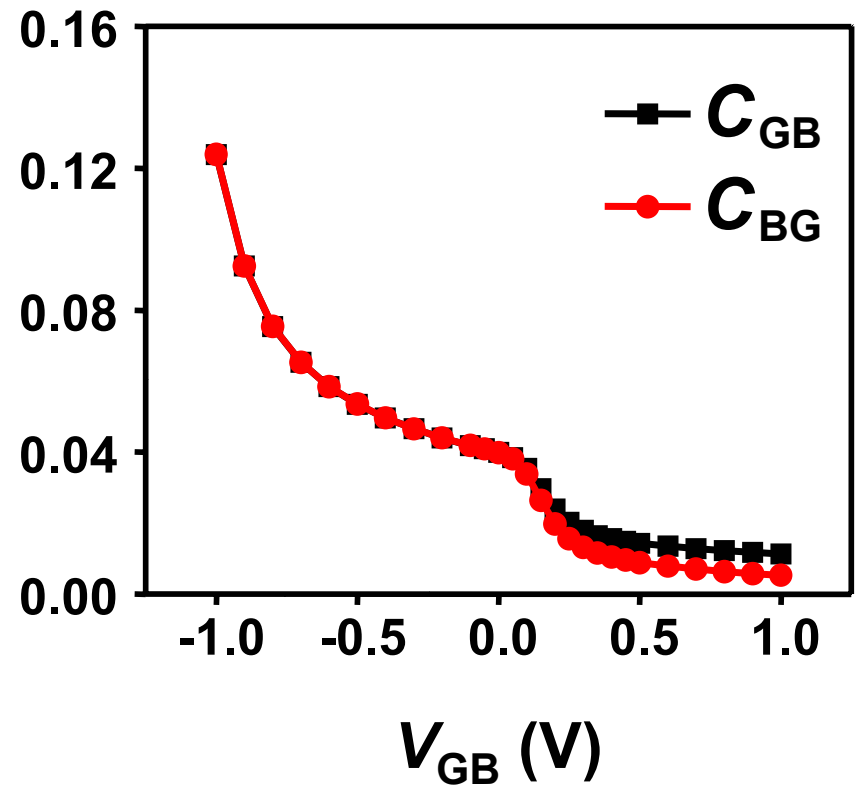
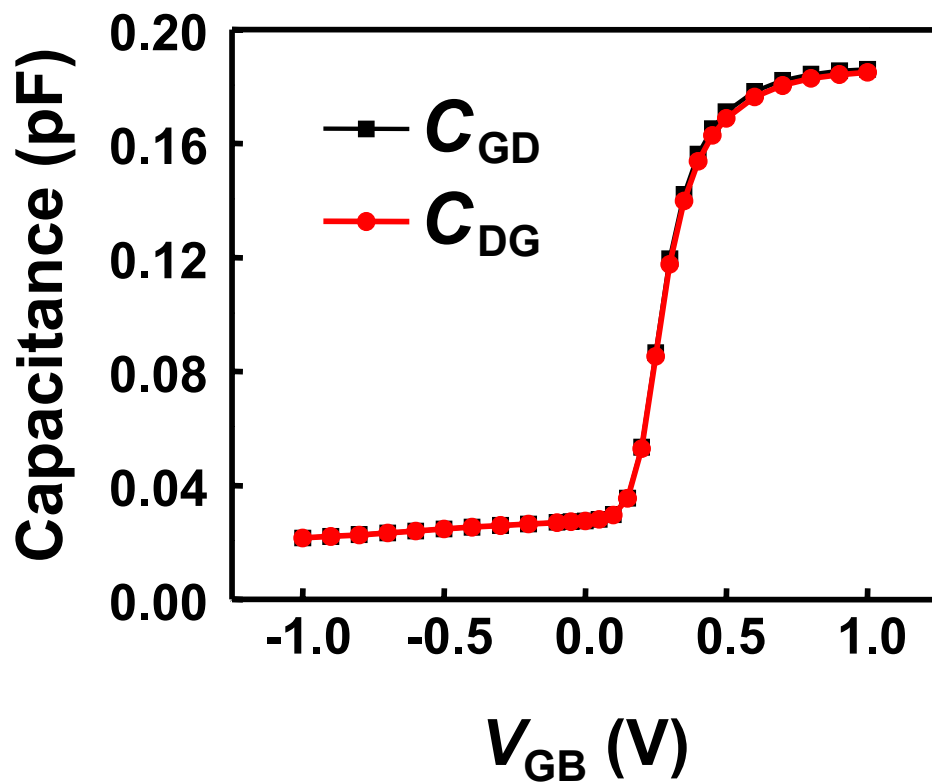
# Reciprocity Requirement

- General speaking,  $C_{ij} \neq C_{ji}$ . But for the special case of  $V_{DS}=0$ ,  $C_{ij} = C_{ji}$ . For example,  $C_{GD} = C_{DG}$  and  $C_{GB} = C_{BG}$

This requirement is rather subtle (violated in most models) Satisfied in PSP 101.0 to the extend allowed by the charge-sheet approximation (1% of  $C_{ox}$  or better)

- Complete symmetry of PSP implies that for  $V_{DS}=0$ ,  $C_{SD} = C_{DS}$  exactly

# Reciprocity of Transcapacitances for $V_{DS} = 0$



# Acknowledgement

- ❑ The authors are grateful to P. Bendix, S. Chaudhry, D. Foty, C. McAndrew, B. Mulvaney, J. Victory, S. Veeraraghavan, J. Watson, G. Workman for numerous stimulating discussion of the subject
- ❑ PSP development at PSU was supported in part by SRC, LSI Logic, Freescale Semiconductor, IBM and by simulation tools provided by Freescale Semiconductor, Mentor Graphics and Agilent
- ❑ Evaluation of PSP100.1 by CMC member companies resulted in significant improvements of the model reflected in PSP101.0 – new industry standard

# Conclusions (I)

- ❑ The results of surface potential calculations in PSP are indistinguishable from the numerical solutions for all bias conditions
- ❑ Symmetric linearization method is not limited to any particular velocity-field relation
- ❑ Saturation region modeling in PSP is highly accurate including higher order output conductances

# Conclusions (II)

- ❑ Mobility models in advanced compact models must include Coulomb scattering. “Universal” models no longer suffice. Higher order transconductances are well reproduced in PSP
- ❑ Retrograde doping is included
- ❑ PSP satisfies the reciprocity requirement at zero drain bias
- ❑ PSP has been tested down to 65nm nodes
- ❑ PSP passes both the old and the new symmetry tests