A Charge-based Compact Model of Double Gate MOSFET

A. S. Roy[1] C. C. Enz[1,2] J. M. Sallese[1]

[1]Ecole Polytechnique Fédérale de Lausanne (EPFL)
Laboratoire d'Electronique Générale (LEG), CH-1015 Lausanne, Switzerland

[2]Swiss Center for Electronics and Microtechnology (CSEM)
CH-2007 Neuchâtel, Switzerland.

Workshop on Compact Modeling, 2006

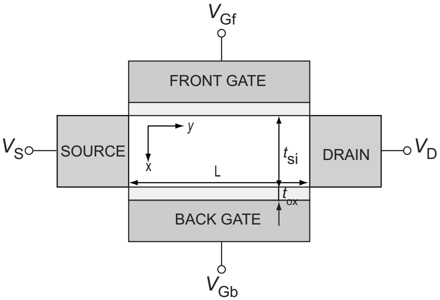
Outline

Introduction

- DG MOST has several intrinsic advantages over bulk MOST
  (i.e. almost ideal subthreshold slope factor, higher on-to-off current ratio, reduced SCEs,...)
- A good compact model is a prerequisite to actually exploit those advantages in IC design
- To date, a simple physical compact model is still lacking even for the ‘classical’ asymmetric DG MOST
- Numerical integration needs solution of a coupled nonlinear equation at each point along the channel
- We present a semi-empirical analytic closed form charge-based expression for the drain current for both symmetrical and asymmetrical DG MOST

Structure and Assumptions

- Ideal structure of long-channel DG MOSFET



- Main assumptions:
  - Long-channel and hence gradual channel approximation (GCA) applies
  - Channel voltage V is constant along the vertical direction x

Normalization

- Model uses the conventional EKV normalization
- Voltages are normalized to the thermodynamic voltage $U_T \triangleq kT/q$

$$v \triangleq \frac{V}{U_T}$$

- Drain current is normalized to the specific current $I_{spec} \triangleq \mu C_{ox} W/L U_T^2$

$$i \triangleq \frac{I}{I_{spec}}$$

- Charges are normalized to the specific charge $Q_{spec} \triangleq C_{ox} U_T$

$$q \triangleq \frac{Q}{Q_{spec}}$$

- Vertical position is normalized to Si thickness t_{si}

$$\xi \triangleq \frac{x}{t_{si}}$$

Drain Current

- The normalized drain current is given by

$$i = \int_{v_s}^{v_d} q_i \mathrm{d}v = \int_{q_s}^{q_d} q_i \frac{\mathrm{d}v}{\mathrm{d}q_i} \mathrm{d}q_i$$

  where
  - $q_i \triangleq q_f + q_b$: total normalized inversion charge density
  - $q_{f(b)}$: normalized inversion charge density at the front (back) gate
  - $q_{s(d)}$: total normalized inversion charge density at the source (drain)
  - v: normalized channel voltage
- Only required information to derive the current is

$$\frac{\mathrm{d}v}{\mathrm{d}q_i}$$

- Will be derived in two steps:
  - Find a relation between the inversion charge and the gate voltages exploiting the front-back gate symmetry
  - Solve the equilibrium electrostatics in different bias situations

Symmetry of Asymmetric DG

- Potential in the channel at equilibrium (assuming GCA)

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} = -\rho = f(\Psi, x)$$

- In presence of channel voltage V

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} = f(\Psi - V, x)$$

- Assuming the channel voltage V does not change along x, $\widetilde{\Psi} \triangleq \Psi - V$ satisfies the equilibrium Poisson equation
- V_G dependence of Ψ comes from the continuity of displacement vector at both silicon oxide interface
- V_G always comes with $V_G - \Psi = V_G - V - \widetilde{\Psi}$
- $\widetilde{\Psi}$ therefore sees $V_G - V$ instead of V_G

Nonequilibrium Potential from Equilibrium

- In equilibrium, Ψ has a form $\Psi_{eq}(V_{Gf}^*, V_{Gb}^*)$ where $V_{Gf(b)}^* \triangleq V_{Gf(b)} - \Delta_{f(b)}$ is the effective front (back) voltage including work function difference $\Delta_{f(b)}$
- $\widetilde{\Psi}$ is then given by

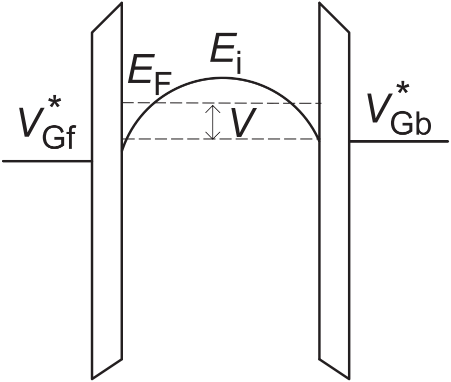
$$\widetilde{\Psi}(V_{Gf}^*, V_{Gb}^*, V) = \Psi_{eq}(V_{Gf}^* - V, V_{Gb}^* - V)$$

- Potential in nonequilibrium is obtained from the definition of $\widetilde{\Psi}$ as

$$\Psi(V_{Gf}^*, V_{Gb}^*, V) = V + \Psi_{eq}(V_{Gf}^* - V, V_{Gb}^* - V)$$

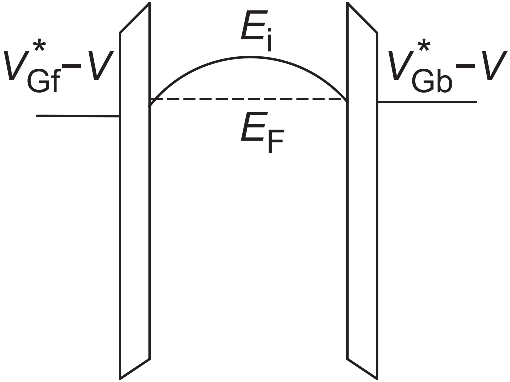
- Nonequilibrium potential at a point x is obtained from the equilibrium potential increased by V and with the gate voltages reduced by V

Illustration of Symmetry in the Energy Bands



- Effect of the channel voltage V can therefore be referred to both gates, bringing the channel in an equilibrium situation corresponding to the shifted gate voltages

Illustration of Symmetry in the Energy Bands



- Effect of the channel voltage V can therefore be referred to both gates, bringing the channel in an equilibrium situation corresponding to the shifted gate voltages

Nonequilibrium Charge from Equilibrium

- The front (back) gate charge $Q_{f(b)}$ is given by

$$Q_{f(b)} = C_{ox}(V^*_{Gf(b)} - \Psi_{f(b)})$$

where $\Psi_{f(b)}$ is the potential at the front (back) gate

- The charges Q_f, Q_b and $Q_i \triangleq Q_f + Q_b$ in disequilibrium are then given by

$$Q_x(V^*_{Gf}, V^*_{Gb}, V) = Q_{xeq}(V^*_{Gf} - V, V^*_{Gb} - V) \quad \text{with} \quad x = \{i, f, b\}$$

where Q_{xeq} is the equilibrium charge corresponding to Q_x

Common and Differential Mode Gate Voltages

- From front and back gate symmetry, increasing the channel voltage V is then equivalent to reducing the CM gate voltage V_{CM}, keeping the differential mode gate voltage V_{DM} constant

$$V_{CM} \triangleq \frac{V_{Gf}^* + V_{Gb}^*}{2} \quad \text{and} \quad V_{DM} \triangleq \frac{V_{Gf}^* - V_{Gb}^*}{2}$$

- The charges can now be written as

$$Q_x(V_{Gf}^*, V_{Gb}^*, V) = Q_{xeq}(V_{CM} - V, V_{DM})$$

- From which we have

$$\frac{\mathrm{d}Q_i}{\mathrm{d}V} = -\frac{\mathrm{d}Q_{ieq}}{\mathrm{d}V_{CM}} \tag{1}$$

- To find the current we now have to express $\mathrm{d}Q_{ieq}/\mathrm{d}V_{CM}$ as function of Q_i

Integration of Poisson Equation

- Normalized Poisson equation after integration

$$v_{gf(b)} = q_{f(b)} + \ln\left(q_{f(b)}^2 + 2a^2 C\right) - \ln(2a^2)$$

where a is a technology parameter given by

$$a \triangleq \frac{\varepsilon_{si}}{\varepsilon_{ox}} \frac{t_{ox}}{L_D} \quad \text{where} \quad L_D \triangleq \sqrt{\frac{U_T \varepsilon_{si}}{q n_i}}$$

- C is the integration constant which is independent of x but depends on the gate voltages according to

$$C = e^{\psi} - \frac{1}{2} \frac{L_D^2}{t_{si}^2} \left(\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right)^2$$

- Derive $\mathrm{d}Q_{ieq}/\mathrm{d}V_{CM}$ as function of Q_i in the following asymptotic modes of operations:
  - Both interfaces in weak inversion
  - Only one interface in strong inversion
  - Both interfaces in strong inversion

Both Interfaces in Weak Inversion

- In weak inversion the electric field $E = -\mathrm{d}\Psi/\mathrm{d}x$ can be assumed constant along the vertical direction x
- The inversion charge is then given by

$$Q_i = -\frac{1}{E}\int_{\Psi_f}^{\Psi_b} qn_i e^{\frac{\Psi}{U_T}}\, d\Psi = \frac{qn_i U_T}{E}\left(e^{\frac{\Psi_f}{U_T}} - e^{\frac{\Psi_b}{U_T}}\right).$$

- Capacitive divider since inversion charge small in the silicon

$$E = \frac{2V_{DM}}{t_{si} + 2(\frac{\varepsilon_{si}}{\varepsilon_{ox}})t_{ox}}$$

- In addition, since E is constant along the Si film thickness

$$Q_i = \frac{qn_i U_T}{E} e^{\frac{\Psi_f}{U_T}}\left(1 - e^{\frac{-Et_{si}}{U_T}}\right) = Ke^{\frac{\Psi_f}{U_T}}$$

where K does not depend on the common mode gate voltage

Both Interfaces in Weak Inversion

- For the asymmetric case, in WI, we have in normalized form

$$\frac{\mathrm{d}q_i}{\mathrm{d}v_{cm}} = q_i \frac{\mathrm{d}\psi_f}{\mathrm{d}v_{cm}}$$

- Capacitive divider

$$\psi_f = \frac{C_{si} + C_{ox}}{2C_{si} + C_{ox}} v_{gf}^* + \frac{C_{si}}{2C_{si} + C_{ox}} v_{gb}^* = v_{cm} + \frac{C_{ox}}{2C_{si} + C_{ox}} v_{dm},$$

  where C_{si} is the silicon capacitance
- Therefore $\mathrm{d}\psi_f/\mathrm{d}v_{cm} = 1$ and finally

$$\frac{\mathrm{d}q_i}{\mathrm{d}v_{cm}} = q_i \quad \text{or} \quad \frac{\mathrm{d}v_{cm}}{\mathrm{d}q_i} = \frac{1}{q_i}$$

Only One Interface in Strong Inversion

- The integrated Poisson equation can be written in terms of v_{cm} and v_{dm} as

$$v_{cm} + v_{dm} = q_f + \ln\left(q_f^2 + 2a^2 C\right) - \ln(2a^2)$$

- In order to evaluate $\mathrm{d}q_i/\mathrm{d}v_{cm}$, we have to express q_f and C in terms of q_i, v_{cm} and v_{dm}
- Considering the back gate in WI, C can be approximated by

$$C = e^{\psi} - \frac{1}{2}\frac{L_D^2}{t_{si}^2}\left(\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right)^2 \approx -\frac{1}{2}\left(\frac{L_D}{t_{si}}\right)^2\left(\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right)^2$$

- Assuming charge sheet assumption at the front gate and the fact that volumetric inversion is small, we get

$$\frac{\mathrm{d}\psi}{\mathrm{d}\xi} \simeq \psi_b - \psi_f$$

Only One Interface in Strong Inversion

- Since the back gate is in WI, from the continuity of displacement vector in the back interface, we have

$$\psi_b = \alpha\psi_f + (1-\alpha)v_{gb}^* \quad \text{where} \quad \alpha \triangleq C_{si}/(C_{si}+C_{ox})$$

- We can now define q_f and $\mathrm{d}\psi/\mathrm{d}\xi$ (and hence C) in terms of $q_i = q_f + q_b$

$$q_f = \frac{q_i + 2\alpha v_{dm}}{1+\alpha} \quad \text{and} \quad \frac{\mathrm{d}\psi}{\mathrm{d}\xi} = \frac{1-\alpha}{1+\alpha}(q_i - 2v_{dm})$$

- We now have

$$v_{cm} + v_{dm} = \frac{q_i + 2\alpha v_{dm}}{1+\alpha} + \ln\left(q_i^2 + 4\frac{C_{si}}{C_{ox}}v_{dm}q_i\right) + const.$$

- From which, we get

$$\frac{\mathrm{d}v_{cm}}{\mathrm{d}q_i} = \frac{1}{1+\alpha} + \frac{2q_i + 4\frac{C_{si}}{C_{ox}}v_{dm}}{q_i^2 + 4\frac{C_{si}}{C_{ox}}v_{dm}q_i}$$

Both Interfaces in Strong Inversion

- Normalized Integrated Poisson equation

$$v^*_{gf(b)} = q_{f(b)} + \ln\left(q^2_{f(b)} + 2a^2C\right) + const.$$

- In strong inversion, $2a^2C^2 \ll q_f^2, q_b^2$ and hence

$$v^*_{gf(b)} = q_{f(b)} + 2\ln(q_{f(b)}) + const.$$

- Adding these two equations we get

$$2v_{cm} \approx q_f + q_b + 2\ln(q_f q_b) + const.$$

- Asymptotically we can assume that $q_f \approx q_b \approx q_i/2$ inside the log term

$$2v_{cm} = q_i + 4\ln(q_i) + const.$$

- Which directly leads to

$$\frac{\mathrm{d}v_{cm}}{\mathrm{d}q_i} = \frac{1}{2} + \frac{2}{q_i}$$

Interpolation of Asymptotic Modes

- We now have the following asymptotic situations:
  - Both interfaces in WI:
$$\frac{dv_{cm}}{dq_i} = \frac{1}{q_i}$$
  - Front gate in SI and back gate in WI
$$\frac{dv_{cm}}{dq_i} = \frac{1}{1+\alpha} + \frac{2q_i + 4\frac{C_{si}}{C_{ox}}v_{dm}}{q_i^2 + 4\frac{C_{si}}{C_{ox}}v_{dm}q_i}$$
  - Both interfaces in SI
$$\frac{dv_{cm}}{dq_i} = \frac{1}{2} + \frac{2}{q_i}$$
  - For symmetrical (i.e. $v_{gf} = v_{gb}$ and hence $v_{dm} = 0$)
$$\frac{dv_{cm}}{dq_i} = \frac{1}{2} + \frac{2q_i + 4\frac{C_{si}}{C_{ox}}}{q_i^2 + 4\frac{C_{si}}{C_{ox}}q_i}.$$
- All these modes can be interpolated by an appropriate function using two fitting parameters which are set to fixed values

Closed-Form Expression of Current

- The expression of the normalized current is given by

$$i \triangleq I_D/I_{spec} = i_0(q_s) - i_0(q_d)$$

  where $I_{spec} = \mu C_{ox} \frac{W}{L} U_T^2$ and where $i_0(q_i)$ is given by
- When both interfaces are inverted

$$\begin{aligned} i_0(q_i) = & \frac{q_i^2}{4} + 2q_i \\ & - \frac{2}{\sqrt{2}} \cdot \left(\frac{1}{2} - \frac{1}{1+\alpha}\right) \cdot (v_{dm})^2 \cdot tan^{-1}\left(\frac{q_i^2}{2\sqrt{2}(v_{dm})^2}\right) \\ & - 4\frac{C_{si}}{C_{ox}}(v_{dm}+1)\ln\left(q_i + 4\frac{C_{si}}{C_{ox}}(v_{dm}+1)\right) \end{aligned}$$

- Valid in all modes of operation
- Valid for asymmetric work functions and gate voltages

Validation – Drain Current vs. CM Gate Voltage

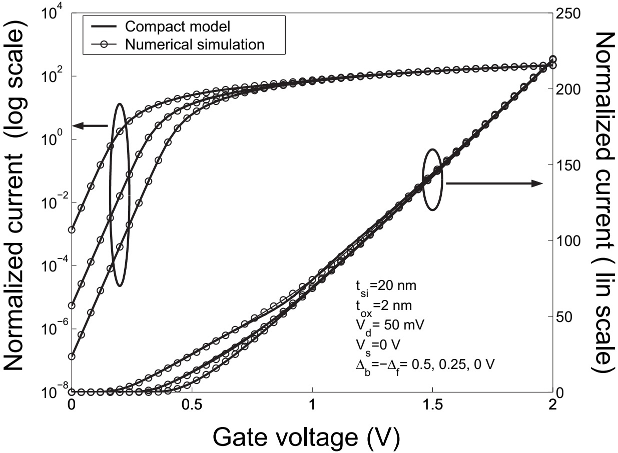


Figure: I_D vs V_G in the linear region for several diff. voltages $V_{DM} = \Delta_b$ = -Δ_f

Validation – Drain Current vs Drain Voltage

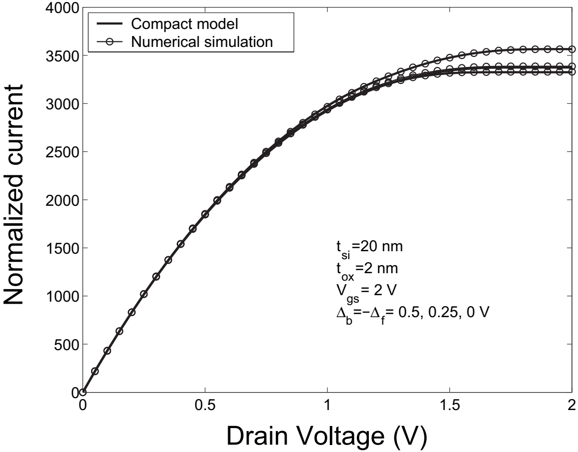


Figure: I_D vs V_D for several differential voltages $V_{DM} = \Delta_b$ = -Δ_f

Validation – Drain Current vs Drain Voltage

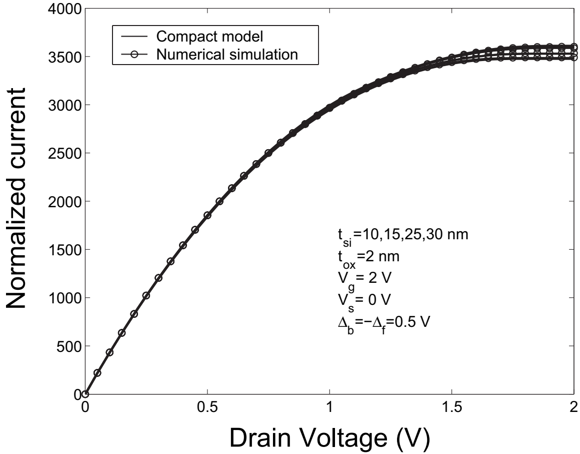


Figure: I_D versus V_D for several Si thicknesses t_{si}

Validation – G_m/I_D Ratio vs Normalized Current

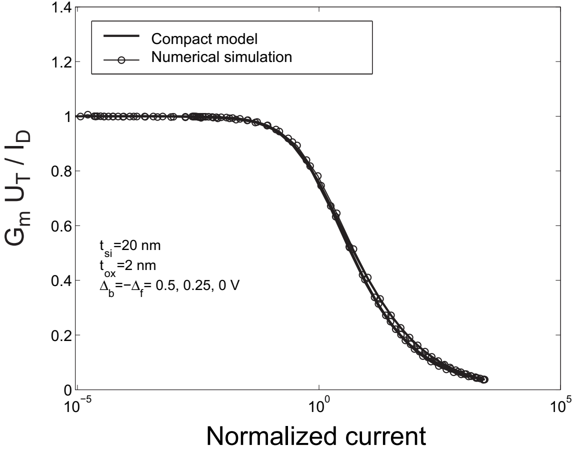


Figure: G_m/I_D ratio vs normalized current in sat. for several $V_{DM} = \Delta_b$ = -Δ_f

Validation – Charge vs CM Gate Voltage

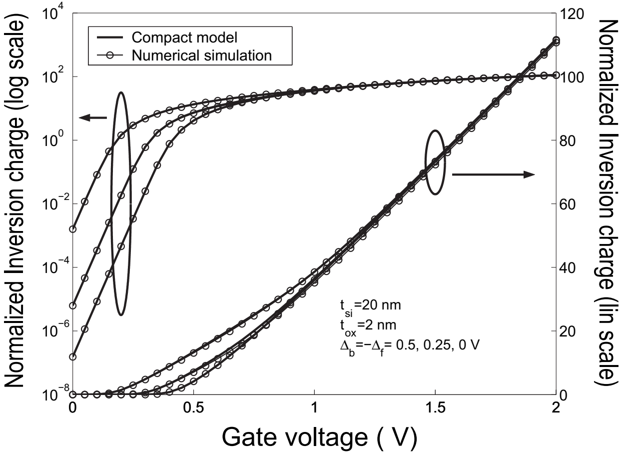


Figure: Inversion charge vs V_G in saturation

Conclusion

- Unique symmetry between common mode gate voltage and channel voltage
- Analytical closed-form charge-based expression for the drain current of asymmetric DG MOST
- Both charge and current are obtained through a coherent picture using only the physical parameters of the device
- A good match with exact numerical solution over a wide range of bias and geometry
- This model can also be extended to small-signal analysis